

## Exercises on Bellman Equation under certainty

1. A particle moves along the  $x$ -axis under a force which is given as  $h(x, \dot{x})$ . Initially  $x(0) = a$ ,  $\dot{x}(0) = b$ . If  $H(a, b)$  is the maximum displacement show that  $H$  obeys the equation

$$b + b \frac{\partial H}{\partial a} + h(a, b) \frac{\partial H}{\partial b} = 0.$$

2. Solve the problem

$$\min \int_0^T (x^2 + \dot{x}^2) dt$$

subject to  $x(0) = x$  using the Euler-Lagrange equation.

3. Find the Bellman Equation when

$$S(c, T) = \min_{x(0)=c} \left\{ \int_0^T (\dot{x}^2 + x^2) dt + x(T)^2 \right\}.$$

4. Solve the maximum height problem when air resistance is included so that  $\ddot{x} = -g - \dot{x}^2$ .

5. Solve the maximum height problem when air resistance is modelled so that the equation of motion is  $\ddot{x} = -1 - \dot{x}^3$ , where units of measurement have been selected so that  $g = 1$ . You will find it useful to note that:

$$\frac{3}{v^3 + 1} = \frac{1}{v + 1} - \frac{v - 2}{v^2 - v + 1}.$$