

Exercises on unrestricted optimal control

1. Find the unrestricted control u which minimizes the cost function

$$\int_0^2 (u^2 + x) dt$$

subject to: $\dot{x} = -x + u$ with $x(0) = 1$. What is the optimal control if the control variable is restricted so that $0 \leq u \leq 2$? Justify your answer.

2. Find the unrestricted control u which minimizes the cost function

$$\int_0^\infty \frac{1}{2} \left(x^2 + \frac{16}{3} u^2 \right) dt$$

subject to: $\ddot{x} = -\dot{x} + u$ with the requirements that $x(0) = 1$, $\dot{x}(0) = 0$, and that $x(t) \rightarrow 0$ and $\dot{x}(t) \rightarrow 0$ as $t \rightarrow \infty$. [Hint: Eliminate the co-state variables in favour of u and x .]

3. Find the unrestricted control u which minimizes the cost function

$$\int_0^1 u^2 dt$$

subject to: $\ddot{x} = u$ with $x(0) = \dot{x}(0) = x(1) = 1$ but with $\dot{x}(1)$ unspecified.

4. Verify that when $x(T)$ is unspecified, the natural boundary condition on the Lagrangian gives $\lambda(T) = 0$.

5. Solve the problem:

$$\min \int_0^1 \left(\frac{1}{2} u^2 - x^2 \right) dt$$

subject to:

$$\dot{x} = x + u, \quad x(0) = 0, \quad x(1) \text{ specified.}$$

What happens when u is constrained? What happens when $x(1)$ is unspecified?

6. Find the unrestricted control u which minimizes the cost function

$$\int_0^1 (x^2 - 2x + u^2) dt$$

subject to: $\dot{x} = u - ax$ with $x(0) = 0$, $x(1) = 1$. [Hint: First find $x(t)$ and use hyperbolic functions.]

7. Solve the unrestricted control problem:

$$\min \int_0^1 (2x^2 + 2xu + u^2) dt$$

with $\dot{x} = x + u$ and $x(0)$ specified and $x(1)$ unspecified.

8. For the problem

$$\min \int_0^T ((x - t)^2 + u^2) dt$$

with $\dot{x} = ax + u$ with a a constant, show that $x = \frac{t}{1 + a^2} + B \sinh(t\sqrt{1 + a^2})$.

9. Extremize $\int_0^1 u^2 dt$ with u unconstrained subject to $\ddot{x} = u$ with $x(0) = \dot{x}(0) = 1$; $x(1), \dot{x}(1)$ not specified.