Some Second Order Differential equations

Just for the record, I've listed below some standard cases from the classical menagerie. Some of these have become relevant in Finance. Information on them is best sought in the following texts:

Handbook of Mathematical Functions by Abramowitz & Stegun,
Whittaker & Watson's *A course of Modern Analysis*,
Lebedev, *Special Functions and their applications*

Parabolic cylinder functions

$$U'' + (ax^2 + bx + c)U = 0$$

with two forms:

$$\begin{aligned} U'' - (\frac{1}{4}x^2 + a)U &= 0, \\ U'' + (\frac{1}{4}x^2 - a)U &= 0. \end{aligned}$$

Hermite

$$U'' - 2xU' + 2nU = 0$$

Chebyshev

$$(1 - x^2)U'' - xU' + n^2U = 0$$

Bessel

$$x^2U'' + xU' + [x^2 - n^2]U = 0$$

Modified Bessel

$$x^2U'' + xU' - [x^2 + n^2]U = 0$$

Legendre:

$$(1 - x^2)U'' - 2xU' + n(n + 1)U = 0$$

Laguerre

$$xU'' + (1 - x)U' + nU = 0$$

Generalized Laguerre

$$xU'' + (\alpha + 1 - x)U' + nU = 0$$

$$xL'_n = nL_n - (n + \alpha)L^{\alpha}_{n-1}$$

Confluent hypergeometric

$$xU'' + (d - x)U' - aU = 0$$

Hypergeometric

$$x(1 - x)U'' + [c - (a + b + 1)x]U' - abU = 0,$$

Riemann's general scheme

$$\begin{aligned} 0 &= U'' + \left[\frac{1-\alpha-\alpha'}{x-a} + \frac{1-\beta-\beta'}{x-b} + \frac{1-\gamma-\gamma'}{x-c}\right] U' + \\ &\left[\frac{\alpha\alpha'(a-b)(a-c)}{x-a} + \frac{\beta\beta'(b-c)(b-a)}{x-b} + \frac{\gamma\gamma'(c-a)(c-b)}{x-c}\right] \frac{1}{(z-a)(z-b)(z-c)} U \end{aligned}$$

Spheroidal Wave equation

$$(1-x^2)U'' - 2(b+1)xU' + [c - 4qx^2]U = 0$$