Inference from non-disclosure

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Seminar papers


Background literature


Other (eg microstructure) literature


Impact of News: Empirical studies


[Cousin, Jean-Gabriel and de Launois, Tanguy, "Investigation of the Impact of the Financial Communication Intensity on the Conditional Volatility of Stock (preprint).]
"Received" wisdom of disclosure literature

Greater disclosure will lead to a lower cost of capital

- However a number of *seemingly* anomalous results eg:
  
  Botosan & Plumlee (2002) find firms that make more timely disclosures face an increased cost of capital relative to others offering less timely disclosures

  Are these anomalies OR do we not have an adequate equilibrium model of disclosure choice?
Information from self-interest

Looking at earnings announcements Shin (2007) argues that:

1. Unlike many sources of public information, such as macroeconomic announcements, earnings disclosures are distinguished by the fact that the information is provided by interested parties. The resolution of uncertainty in such contexts takes on distinctive features that are quite unlike the resolution of uncertainty associated with exogenously generated signals.

2. The distribution of future earnings will play an important role in the determination of asset prices today. To gain the full picture, it seems essential to have an asset pricing framework that incorporates a model of disclosures as an integral part of the overall framework.
Aim

Model the link between cost of capital and disclosure policy

= Find asset pricing framework incorporating disclosure

Extant literature focusses on horizontal asymmetries (between investors)

not on vertical assymmetry (manager vs. investor)

Problem:

Established model of vertical asymmetry (Dye 1985) not amenable to empirical study (lacks observable variables).
Dye’s model

Grossman Milgrom Hart Paradox: Full disclosure in equilibrium.

Managers will voluntarily disclose all information when disclosure is costless because failure to do so will cause participants with rational expectations to infer the minimum valuation (fear the worst about withheld information).

But this is not what is observed.

Dye insight - what if there is a positive probability that the manager has not observed any information to disclose?
\( \text{time} = 0 \) (Ex-ante) Risk-neutral distribution \( F_X \) of the terminal value \( X \)

\( \text{time} = 1 \) (Interim report) Manager see \( X \) with probability \( q = 1 - p \); remains ignorant with prob \( p \).

Manager has option to report observed information credibly (if endowed, but not otherwise).

\( \text{time} = 2 \) (Terminal date) \( X \) becomes known.

1. Assumes that: disclosure is credible

2. Lack of information cannot be communicated credibly
3. If threshold (cutoff) $t$ for reporting of observed news is known as $t$, then investor’s valuation when no disclosure

$$V(t) = E[X|ND(t)]$$

Dye cutoff $\gamma$ given by **Dye Equation**

$$\gamma = V(\gamma).$$

Indifference in equilibrium between disclosing $\gamma$ and not.
Dye equation in distributional format

\[ \frac{p}{1-p}(m_X - \gamma) = H_X(\gamma) := \int_{x \leq \gamma} (\gamma - x) dF_X(x) = H_X(\gamma) := \int_{x \leq \gamma} F_X(x) dx. \]

Here \( H \) stands for Hemi-mean. Note the significant components:

- \( m_X - \gamma \) = market downgrade resulting from non disclosure

and with it the deduction: \( H_X(\gamma) \) represents the value of a potential upgrade.

- \( \lambda = \frac{p}{1-p} \) = information endowment odds for management (a natural variable)
Background: state space

\[ \Omega = \Omega_{\text{type}} \times \Omega_{\text{value}} = \{0, 1\} \times \mathbb{R}_+ \]

where 0 means not endowed with information. For arbitrary cutoff \( t \) we put:

Disclosure event \( D(t) = \{1\} \times [t, \infty) \subset \Omega_{\text{type}} \times \Omega_{\text{value}} \)

Non-disclosure event \( ND(t) = \{0\} \times \mathbb{R}_+ \cup \{1\} \times [0, t] \subset \Omega \).
When a portfolio manager is choosing what stocks to hold "she" gives more weight to downside risk than upside risk i.e. preferences over risk are not well measured using $\sigma^2$ hence a lot of research (since Markowitz) has been concerned with evaluating risks in the lower tail. i.e. given a distribution, assessing events in the lower tail differently than elsewhere. (Lower Partial Moments, e.g. SemiVariance)

What aligns the Dye model with this aim is the presence of:

$$ H_X(\gamma) = \int_{x \leq \gamma} (\gamma - x) dF_X(x) = \text{LPM}(1) $$
Dye model improved

Drawbacks:

1. $p$ is undetermined

2. Observation of $X$ on a preview basis is unrealistic
Solution

1. Endogenize $p$ and let manager choose $\hat{p}$ so as to maximize firm value,

2. Introduce noisy signal $Y$ and let observed signal be

$$T = T(X, Y) = \ldots \text{ eg } = X + Y \text{ or } = XY.$$ 

Replace $X$ by

$$EST[X] := E[X|T(X, Y)]$$

require that the regression function is monotone. Below we may identify $S$ with $X$.

Get: observable disclosure intensity

$$\hat{\tau} = (1 - \hat{p})[1 - F_{EST}(\gamma_{EST}(\hat{p}))]$$
Dealing with noise

An equivalent reformulated problem:

Using $\text{EST}(X) := E[X|T]$ in place of $X$. Let $t$ be a realization of $t$. Dye equilibrium cutoff $\gamma$ is now chosen as a cutoff for $\mu_X(t)$. Equation satisfied by $\gamma$ is now

$$\frac{p}{1-p}(m_X - \gamma_{\text{EST}}(p)) = H_{E[X|T]}(\gamma(p))$$

So below work with $X$, but interpret its variance as $\sigma_{\text{aggregate}}$ – combining sector variability $\sigma_X$ and management variability $\sigma_Y$. 
Dye’s model: a fresh view

Minimum Principle of Valuation

**Theorem.** Put $V(t) = E[X|ND(t)]$ and suppose that the density has $f_X(x) > 0$ and is continuous. Then $V(t)$ has a unique **minimum** at $t = \gamma$. 
Firm /manger’s type – given by $\sigma_Y$ (vision)

Desired monotonicity theorem: $\hat{\tau}_Y$ monotonic in $\sigma_Y$ (To distinguish between firms in a sector with $\sigma_X$ fixed).

Yes, if $F_X$ is log-concave and $F_X$ ‘exhibits risk aversion’ (defn. below).

Monotonicity, but which way? ... increasing ... thus

**Information Sharing Principle:** Higher risk (poorer vision) requires higher $\hat{\tau}$,

i.e. more sharing of information with the market.
A Maximum Principle

(Simple buy-out Scenario)

Permit manager to trade on lack of information. Then in Kyle style ‘unobserved’ selling

$$V_{\text{manager}}(p, t) := p(m_X - V(t)),$$

as Manager uninformed with probability $p$ and firm value is $E[X]$ rather than $V(t)$.

Manager's option is maximized with $t = \gamma(p)$.

Other trading mechanisms possible; with optimal randomized trading these yield concave $V_{\text{manager}}(p)$. 
Trade-off implications: the implied utility

Simple buy-out case

\[ p(m_X - \gamma) = (1 - p)H_X(\gamma) \]

risk-shield effect (value protection) = value enhancement opportunity.

Put

\[ x := m - \gamma, \quad y := H_X(m - x) \]
\[ px = (1 - p)y \quad \text{so} \quad p = \frac{y}{x + y} \]

Maximized Objective

\[ p(m - \gamma) = px = \frac{xy}{x + y} = (x^{-1} + y^{-1})^{-1} \]

yielding an *implied utility* of the CES type.
Figure 1. The arbitrage line (blue), the opportunity curve (red), and the tangential utility contour (green).
Optimality Condition / tangency condition

Theorem

(i) \( \hat{\lambda} = \sqrt{F(\hat{\gamma})} \) and (ii) \( \tau = 1 - \lambda \) iff \( \tau = \hat{\tau} \).

These equilibrium conditions are driven by the trading mechanisms (other mechanisms will shift \( \gamma \)).

Note that

\[ \hat{p} < \frac{1}{2}, \]

so ‘more endowed than unendowed’.
Trade-off implications

General case

More general trading models (allowing for strategic effects) yields valuation

\[ V(p) \cdot (m - \gamma(p)) \]

\[ U(x, y) := V \left( \frac{y}{x + y} \right) x, \quad \text{since } p = \frac{y}{x + y} \]

is the corresponding implied utility

NB: This is homothetic, i.e a function of \( \frac{y}{x} = \frac{p}{1 - p} \)

Reduced problem: optimal choice of \( p \) given by trade-off between loss and gain:
\[ \arg \max_p U(m_X - \gamma(p), H_X(\gamma(p))) \]

- concave in \( p \) so unique optimum exists: below use homothetic \( U \).
Good news

No different to the simple buy-out case

Technical analysis not much different provided $U$ is homothetic or homogeneous of degree 0. In such cases the mathematics depends on $u(\lambda) :=$ Marignal Rate of Substitution of $U$ as a function of the odds $\lambda$. 

$$u(\hat{\lambda}) = F(\hat{\gamma})$$
Theorem (Monotonicity Theorem: Disclosure response to optimal odds). The intensity of disclosure as a function of the optimal odds is decreasing in the following two circumstances:

(i) If \( u(\lambda) \) is increasing, then \( \tau(\lambda) \) is decreasing for all \( \lambda > 0 \).

(ii) If \( u(\lambda) \) is convex and \( \tau'(0) < 0 \), then \( \tau'(\lambda) < 0 \) for all \( \lambda > 0 \).

(ii) If \( u(\lambda) \) is concave and \( \tau'(\bar{\lambda}) < 0 \), then \( \tau'(\lambda) < 0 \) for all \( 0 < \lambda < \bar{\lambda} \).

This is ‘instead’ of

\[
(i) \hat{\lambda} = \sqrt{F(\hat{\gamma})} \quad \text{and} \quad (ii) \tau = 1 - \lambda \text{ iff } \tau = \hat{\tau}.
\]
A little intuition for choice over $p$

For $u(\lambda) := \text{MRS of } U \text{ (homothetic)} = \lambda^\delta$ is power function then in this special case optimal $p$ such that

$$0 < p < 0.5$$

i.e. more informed than uninformed!

when $p = 0$ this is the limiting case where the manager always sees a realization of $X$ and $\gamma = x$.

in the limit when $p = 0.5$ this tells us that the most uninformed the manager will ever become is having a 50% chance of not observing the realization, with implications for possible principal-agent model extension
Modelling risk preferences

A family of distributions $F(x, \sigma)$ one should respect the fact that investors require to be rewarded when accepting increased risk. Our view: investors measure the risk by the gain-to-loss ratio, defined in response to a $\sigma = \sigma_\mu$ and a (freely chosen) cutoff $\gamma$ by

$$\Lambda = \Lambda(\gamma, \sigma) := \frac{H(\gamma, \sigma)}{m_X - \gamma}.$$  

Here $H$ refers either to $H_X$ when the observed signal is that of true value $X$, or else its correction $H^L_T$ (for which see Appendix 1). The latter corresponds to the observed signal being $T = T(X, Y)$ and the true value is estimated by the manager as $\mu_X(T)$.

In fact, of course, since here $\gamma$ is fixed only $H(\gamma, \sigma)$ varies with $\sigma$. 
Assumption MIP (Monotonic Investor Preferences). We assume that for any cutoff $\gamma$ an investor facing an increase in $\sigma = \sigma_\mu$ demands a higher value of $\Lambda(\gamma, \sigma)$, equivalently a higher value of $H(\gamma, \sigma)$, to reward the extra risk-exposure.
Investor risk preference

Suppose that $F(x, \sigma)$ are FSD ordered with mean $m$ and variance $\sigma$. In the simple buy-out case, put:

$$\Pi(x, \sigma) = \sqrt{F(x, \sigma)},$$
$$\Lambda(x, \sigma) = \frac{H(x, \sigma)}{m - x}.$$

MIP holds (i.e. $\hat{\lambda}$ rises with $\sigma$) if

$$\frac{d(\pi, \lambda)}{d(\gamma, \sigma)} = \begin{bmatrix} \pi_\gamma & \pi_\sigma \\ \lambda_\gamma & \lambda_\sigma \end{bmatrix}$$

has positive determinant.
The conditions are met for the log-normal model with $X, Y$ log-normal and

$$T = XY.$$
Result of this analysis:

Lognormal Example: let $T = XY$ with $X = m_X e^{U - \frac{1}{2} \sigma_U^2}$ and $Y = e^{V - \frac{1}{2} \sigma_V^2}$ with $U, V$ independent, normal zero-mean random variables with variances $\sigma_U^2$ and $\sigma_V^2$.

Here $T = m_X e^{W - \frac{1}{2} \sigma_W^2}$ with $W = U + V$ a mean-zero normal with variance $\sigma_W^2 = \sigma_U^2 + \sigma_V^2$ (aggregate variance).

Conditional expectation estimator

$$\text{EST}(X) = E[X|T] = m_X \exp \left( \kappa W - \frac{1}{2} \kappa^2 \sigma_W^2 \right) = m_X \exp \left( \kappa W - \frac{1}{2} \kappa \sigma_U^2 \right),$$
where

\[ \kappa := \frac{\sigma_U^2}{\sigma_U^2 + \sigma_V^2} = \frac{\rho_V}{\rho_U + \rho_V}, \]

employing the precision \( \rho_U = 1/\sigma_U^2 \), etc.
A bounty of Monotonicities:

**Monotonicity of the price of risk-bearing:** For arbitrary fixed $\gamma$ the gain $H(\gamma, \sigma)$ should increase as $\sigma$ increases,

i.e. $H(\gamma, \sigma_1) \leq H(\gamma, \sigma_2)$ for. $\sigma_1 < \sigma_2$.

nb: FSD background: $F(\gamma, \sigma_1) \leq F(\gamma, \sigma_2)$ implies $\bar{F}(\gamma, \sigma_1) \geq \bar{F}(\gamma, \sigma_2)$ (greater probability of disclosure).

* $\lambda$–Odds Monotonicity So, from Dye’s equation: higher $\sigma_W$ yields higher odds and so higher $p$. 
* Monotonicity of aggregate variability: refer to $\kappa^2 \sigma^2_W$ as

$$\sigma^2_{\text{aggregate}} = \kappa^2 \sigma^2_W = \frac{\sigma^4_U}{\sigma^2_U + \sigma^2_V}.$$ 

Note: $\sigma^2_{\text{aggregate}}$ increases with $\sigma^2_U$ and decreases with $\sigma^2_V$.

* $\tilde{\gamma}$–Cutoff Monotonicity Theorem: $\tilde{\gamma}$ does the opposite to $\sigma^2_{\text{aggregate}}$.

* $\hat{\gamma}$–Intensity Monotonicity Theorem: intensity $\hat{\gamma}$ decreasing in $\sigma^2_{\text{aggregate}}$. 
1) inter-industry effect: $\tau$ decreasing in $\sigma^2_U$ for $\sigma^2_V$ fixed (nominal)

*the disclosure intensity falls as a given manager switches to a more volatile industry*

the sector speaks for itself – the manager’s role and so communication less vital

2) intra-industry effect: $\tau$ increasing in $\sigma^2_V$ for $\sigma^2_U$ fixed (nominal): i.e.

*the less precise the manager’s signal the higher the manager’s observed disclosure intensity*

The manager must speak for himself : he is vital. She the investor must be reassured.
(Note holding $m_X$ constant.)

Valid for a wide class of distributions (log concave) subject to positive Jacobian.
Caveats

Truthful disclosure; lack of credible announcement of absence of information.

The model is essentially a single period project model in which success in one period does not influence successes in later periods. So: multi-period project dependence (and related disclosure) is not modelled.

Managers make disclosures according to their own optimal cutoff – no mimicking of a different managerial type; any other behaviour would require alterations to the model to permit this.

We note that the model is robust to changes in the valuation model to other (differentiable) concave valuations: a small perturbation of our chosen value function is reflected in small perturbations elsewhere in the analysis.
Where are we:

Penno 1997 result says (in the Normal case) there is no relationship between $\tau$ and $\sigma^2_V$.

Our result is for a wide class of distributions there is a well defined (monotonic) relationship but BEWARE the two sources of uncertainty have countervailing effects and so any empirical estimation procedure must take this into account.

$\Rightarrow$ v. important when estimating panel to allow industry effects and signal effects to go in opposite directions
Summary

Disclosure-based Asset Pricing Model

I. Technical features

Endogenize $p$ (extends Dye)

Permit (general) noisy observation (extends Penno)
provide new tools:

Minimum Principle of Valuation

Statistical inference via Monotonicity

Information Sharing Principle

Incorporates asymmetric information games (connection with Aumann Maschler theory)
II. Qualitative features

- Penno 1997 result unnecessarily pessimistic

- In reality disclosure is well behaved but in a more complex fashion than initially assumed

- Clearly $\tau$ is not the only empirically observable disclosure variable so this is the start not the end of refining empirical estimation procedures

- Dye model plus endogenous choice of $p$ leads to enhanced understanding of disclosure equilibrium (Shin)
- Potential improvement over CAPM based models in which no role for equilibrium partial disclosure

- PIN versus DIN concentrates on a different source of asymmetry
**Background Details: interpreting $H$**

\[
E[\text{upgrade rel. initial price } m_X] \\
= \int_{x \geq \gamma} (x - m_X) dF_X(x) \\
= \int_{x \leq \gamma} (m_X - x) dF_X(x) \quad \text{(by definition of } m_X = E[X]) \\
= (m_X - \gamma) F(\gamma) + \int_{u \leq \gamma} F_X(u) du, \quad \text{(integration by parts)} \\
= (m_X - \gamma) F(\gamma) + H_X(\gamma), \quad \text{(also integration by parts)} \\
= \text{risk-shield + gain!}
\]
Background: Expected Utility vs Utility of the Expected

Since we are concerned with optimal choice of $p$ how are preferences (utility) defined in such a world?

Fishburn (1977) developed a very general risk-measure (for below-target $t$ risk) which he called the $(\alpha, t)$-model, namely

$$F_\alpha(t) := \int_{\leq t} (t - x)^\alpha dF(x), \quad (\alpha > 0).$$

(with $t$ an exogenous target), and showed it to be tractable. Note $\alpha = 1$ is Dye model.

Fishburn studied preferences over distributions $F$ representable by a utility $U(\mu(F), \rho(F))$ over two parameters associated with $F$: the mean $\mu(F)$ and a risk-measure $\rho(F)$ (instead of $\sigma^2$)
Fishburn derived a necessary and sufficient condition for the equivalence on the one hand of ranking distributions over outcomes by vNM expected utility (over the same outcomes) and on the other hand of ranking distributions $F$ by reference to a utility function $U(\mu(F), \rho(F))$ over two parameters associated with $F$. Here $\mu(F)$ is the mean, and $\rho(F)$ is a risk-measure of the general format $\int_{\leq t} \varphi(t - x)dF(x)$ for $\varphi$ non-negative, non-decreasing with $\varphi(0) = 0$.

Equivalence of expected utility with utility of expectations, i.e. of the two expected values $\mu(F)$ and $\rho(F)$

The latter captures notions of ‘riskiness’ for outcomes $x$ below the target $t$.

Informally Fishburn result = use $\varphi$ (below target) to create a ‘kinked’ (at the target) Utility function.
FSD Preference order over lotteries

Let $F(x, \sigma)$ be a family of distributions with common mean. Each $\sigma$ yields a lottery and $\sigma$ is the variance of $X_\sigma$ and suppose you win the lottery iff $X > t$.

Recall

$$\sigma_1 < \sigma_s \text{ iff } F(x, \sigma_1) < F(x, \sigma_2) \text{ for all } x$$

$$\sigma_1 < \sigma_s \text{ iff } \bar{F}(x, \sigma_1) > \bar{F}(x, \sigma_2)$$

captures the feature that a lottery parametrized by $\sigma_1$ is preferred under FSD (first-order stoch. dominance)
Basis for empirical study: voluntary disclosure

Assume that news is broadcast at a Poisson rate $\theta_V$ dependent on the endowment state $(i, t)$ thus:

$$\theta_V = \begin{cases} 
\alpha t + \beta, & \text{if } e = 1 & t \geq \gamma, \text{ (Disclosure)} \\
\alpha \gamma + \beta, & \text{if } e = 1 & t < \gamma, \text{ or if } e = 0. \text{ (Non-Disclosure)}
\end{cases}$$

Thus $\theta_V$ follows the same constant regime in the non-disclosure region in the outcome space $\{u, i\} \times \mathbb{R}_+$ of the voluntary random variable.
Basis for empirics: mandatory disclosure

Mandatory news treated in similar fashion: assume a lower-threshold (fall in value) for a state variable $Z$, namely $\zeta$ (which precipitates intensive disclosure activity), with $Z$ modelled exogenously. Thus $\theta_M$ follows one of two (state-dependent) regimes:

$$\theta_M = \begin{cases} 
    a - bz & \text{if } z < \zeta, \text{ (bad news)} \\
    a - b\zeta = \delta & \text{if } z \geq \zeta, \text{ (good news)}. 
\end{cases}$$

Conditioning on 'good news' the total Poisson rate of news-wire output

$$N = N_V + N_M$$

is $\alpha t + \beta + \delta$.

Let bars denote conditional expectations:

$$\tilde{N} = \alpha \tilde{T} + \beta + \delta,$$

$$\tilde{\sigma} = \alpha \tilde{\sigma}_T.$$
Given realization $N$, compute score statistic $N^*$ as

$$N^* = \frac{N - \tilde{N}}{\tilde{\sigma}} = \frac{T - \tilde{T}}{\tilde{\sigma}_T},$$

$N^*$ (in the Good News region) is independent of the signalling parameters $\alpha, \beta, \delta$; suggest that this be the basis for empirical analysis.
No-arbitrage basis for $\gamma$

\[
\lambda(m_X - \gamma) = H_X(\gamma) := \int_{x \leq \gamma} F_X(x)dx, \quad \text{(Dye equation)}
\]

\[
\lambda = p/(1-p), \quad \text{odds}
\]

\[
H(\gamma) = \int_{x \leq \gamma} (\gamma - x)dF(x), \quad \text{lower partial moment,}
\]

(Has been used to measures shortfall.) Here $F_X$ regarded as risk-neutral valuation measure.

Dye equation is equivalent to

av. fall under $ND(\gamma) +$ av. rise under $D(\gamma) = 0.$
For any $t$ with $0 < t < 1$, define the claim $C_t$ by the payoff:

$$
C_t(\omega) = \begin{cases} 
  t, & \text{if } \omega \in ND(t), \\
  x, & \text{if } \omega = (i, x) \in D(t).
\end{cases}
$$

**Theorem** For $\mathbb{P}$ the product measure on $\Omega$ built from $F_X$ and $(p, 1 - p)$

$$
E_{\mathbb{P}}[C_t] = m_X \text{ iff } t = \gamma.
$$

For a graphical interpretation, restrict attention to $\{i\} \times \Omega_{value}$ identifying this set with $[0, 1]$, thus obtaining the following claim by restriction to $\Omega_{value}$:

$$
C_t(x) = \max\{X(x), t\} = \begin{cases} 
  t, & \text{if } x < t, \\
  x, & \text{if } x \geq t,
\end{cases}
$$
Figure 1. Call-like valuation $C_\gamma(x)$ versus realized value $x$ (thick); valuation conditional on non-disclosure $V(t)$ versus cutoff $t$ (faint).