432018 PHILOSOPHY OF PHYSICS (Spring 2002)

Lecture 4: The postulates of QM and the Copenhagen interpretation

Reading: Preliminary reading: Sklar, pp. 172-9.

We now turn our attention to the postulates of QM as formulated by von Neumann and we shall discuss these in terms of the formalism devised by Heisenberg. To do this we will start by listing the 'core' postulates of QM which are commonly accepted and then we shall introduce the last, and most controversial postulate. We shall then outline how the formalism is generally used, starting with a general schema for its application and then [briefly] considering an actual example which will allow us to describe the results of the Stern-Gerlach experiments discussed in the previous lecture. Then, we shall consider the 'orthodox' interpretation of QM, namely the Copenhagen interpretation of Bohr and Heisenberg, since this will provide a context for our discussion of more recent interpretations in the next few lectures.

NOTE: It is not necessary for you to fully understand the mathematical aspects of the material covered in this lecture. You just need to understand the basic physical and philosophical points that are being made.

1 The 'core' postulates of QM

Quantum Mechanics is usually formulated in terms of the following, commonly accepted, postulates:

1. **Physical states**: The state of a system allows us to represent what the system is doing at a given time. In QM, we represent them in the following way:

   - Every physical *system* is associated with a particular Hilbert *space*.
   - The possible physical *states* of the system correspond to [normalised, or unit,] vectors in its associated Hilbert space, these are denoted by 'kets', e.g. $|A\rangle$ represents the state A. The state of the system is *completely* represented by the corresponding vector.
   - Every vector picks out a state and every state corresponds to a vector.
   - *Superpositions* of states are themselves states of the system, i.e. any linear combination of vectors is also a vector and as such represents a state.

2. **Measurable properties**: Observables, i.e. properties of the system that we can find by measurement, are represented by linear Hermitian operators acting on the Hilbert space. In particular:

   - If the state $|A\rangle$ is an *eigenstate* of an Hermitian operator $\hat{A}$, i.e. $\hat{A}|A\rangle = a|A\rangle$ where $a \in \mathbb{R}$ is the corresponding *eigenvalue*, then we say that a measurement on the system which is in state $|A\rangle$ yields the value a when me measure the property of the system represented by the operator $\hat{A}$.[1]
   - If we have a superposition of two states, say $|V\rangle$ and $|W\rangle$, then we have

     $$\hat{A}\left(\alpha|V\rangle + \beta|W\rangle\right) = \alpha\hat{A}|V\rangle + \beta\hat{A}|W\rangle,$$

     since the operators we use to represent observables are linear.

3. **Dynamics**: The dynamics describes how the state of a system changes with time.

   - Given the vector that represents the state of the system and information about how it is interacting with other systems, the dynamics allows us to calculate the state of the system at a *later* time.

[1] Notice that we choose Hermitian operators to represent observable properties since they always give real values as the result of a measurement.

- This evolution is governed by Schrödinger's equation, and as such this evolution is *continuous*, *causal* and *deterministic* — i.e. given the state and the interactions there is only one state that the system can evolve to at a later time and it gets there in a continuous manner.

4. **Connection with experiment**: When we perform an experiment, we expect to get different outcomes with different probabilities. This comes into the formalism in the following way:

   - If the system is in a state represented by the vector $|A\rangle$ and this is an eigenstate of the operator $\hat{A}$ used to represent the measurement, then we can predict with *certainty* the outcome of the measurement. That is, if $\hat{A}|A\rangle = a|A\rangle$, then the outcome of the measurement will [certainly] be a.[2]
   - If a system is in a state represented by the vector $|V\rangle$ and this is *not* an eigenstate of the operator $\hat{A}$ used to represent the measurement, then we can find the *probability* that the outcome of the experiment will be a given eigenvalue (corresponding to an eigenstate of the system) using *Born's rule*.[3]

2 Von Neumann's 'projection' (or 'collapse') postulate

The postulates for QM that we saw above can't be the whole story about how the states of a system change over time. Mainly because, as we will see when we discuss our earlier Stern-Gerlach experiments, making a measurement appears to cause the state of the system to change *discontinuously*. That is, if prior to a measurement the system is in a superposition of the relevant eigenstates, then on measurement we will find that the state of the system has 'collapsed' so that it is, in fact, found to be in *only one* of the eigenstates that were in the initial superposition. Indeed, not only does this change of state upon measurement seem discontinuous, it also appears to be indeterministic since we only have the *probability* that a given outcome will manifest itself as a result of such a measurement. But, unfortunately, this kind of discontinuous and random behaviour can *not* be described by the Schrödinger equation (see Postulate 3). So, let's see how von Neumann tried to resolve this problem by invoking a further postulate in his formalism of QM.

Basically, in von Neumann's formalism, the Schrödinger equation (i.e. Postulate 3) only applies when *no* measurements are being made. The effect of a measurement is then captured in the following postulate:

5. **Collapse**: The effect of a measurement is to *change* the state of the measured system. This change is normally called *collapse* since the state changes discontinuously into an eigenstate of the operator representing the observable being measured. Moreover, this collapse is the only place where *randomness* (or 'pure chance') enters into the formalism, i.e. collapse is the only mechanism by which the state vector of the system can evolve probabilistically.

Thus, the collapse postulate *accommodates* the randomness which the probabilities in Postulate 4 are supposed to describe. That is, notice that, unlike the other four postulates there is nothing here for us to calculate.

3 How QM is done I — a schema

In summary then, a quantum mechanical calculation normally proceeds as follows:

[2] And, as such, the post-measurement state will still be $|A\rangle$.

[3] In particular, we can decompose the state $|V\rangle$ in terms of the eigenstates $|A_i\rangle$ of the operator $\hat{A}$, i.e. if

$$|V\rangle = \sum_{i=1}^{n} c_i |A_i\rangle,$$

where for each i we have assumed that

$$\hat{A}|A_i\rangle = a_i|A_i\rangle,$$

then the probability that the measurement will yield the outcome a_i is given by $|c_i|^2$.

- Identify the Hilbert space associated with the system in question and then associate operators with the measurable properties. (Postulates 1 and 2.)
- Given the present state of the system, the *dynamics* (i.e. the Schrödinger equation) gives us a way of calculating the state at any future time. Note that this evolution is continuous, causal and deterministic. (Postulate 3.)
- Using Born's rule, we can calculate the probabilities associated with the different outcomes of a measurement. (Postulate 4.)
- The *effect* of such a measurement on the system, i.e. the discontinuous, indeterministic and seemingly non-causal collapse of the state into one of the eigenstates of the operator representing the measurement, is *accommodated* by the collapse principle. (Postulate 5.)

Let's now see how the formalism and this schema allow us to describe the results of our Stern-Gerlach experiments from the previous lecture.

4 How QM is done II — an example

So, we start by looking at the schema, i.e. we see what we will need to describe the Stern-Gerlach experiments from the previous lecture.

Setting up — Postulates 1, 2 and 3

We start by [briefly] identifying the Hilbert space and the operators which are associated with the property that we are trying to measure with the Stern-Gerlach experiments, namely spin in either the x or y-direction. To do this, we need to find the relevant operators and their eigenstates, i.e.

- An SGX-box measures the x-spin of an electron and the operator representing this property is denoted by $\hat{S}_x$. The eigenstates of this operator are what we have been calling up-x-spin and down-x-spin and we denote these by $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$ respectively. When we make a measurement of the x-spin of an electron which is in the former of these two eigenstates the formalism tells us that
$$\hat{S}_x|\uparrow_x\rangle = +\frac{\hbar}{2}|\uparrow_x\rangle,$$
where $\hbar/2$ is the eigenvalue of this operator corresponding to this state, whereas if the electron is in the latter of these two states we have
$$\hat{S}_x|\downarrow_x\rangle = -\frac{\hbar}{2}|\downarrow_x\rangle,$$
where $-\hbar/2$ is the eigenvalue of this operator corresponding to this state.
- Similarly, an SGY-box measures the y-spin of an electron and the operator representing this property is denoted by $\hat{S}_y$. The eigenstates of this operator are what we have been calling up-y-spin and down-y-spin and we denote these by $|\uparrow_y\rangle$ and $|\downarrow_y\rangle$ respectively. When we make a measurement of the y-spin of an electron which is in the former of these two eigenstates the formalism tells us that
$$\hat{S}_y|\uparrow_y\rangle = +\frac{\hbar}{2}|\uparrow_y\rangle,$$
where $\hbar/2$ is the eigenvalue of this operator corresponding to this state, whereas if the electron is in the latter of these two states we have
$$\hat{S}_y|\downarrow_y\rangle = -\frac{\hbar}{2}|\downarrow_y\rangle,$$
where $-\hbar/2$ is the eigenvalue of this operator corresponding to this state.

Think of an eigenvalue as the *value* of the spin when it is in a particular eigenstate. That is, when we have an electron in the state $|\uparrow_x\rangle$, its spin in the x-direction is given by the number $\frac{\hbar}{2}$ in the appropriate units. Similarly, an electron in the state $|\downarrow_x\rangle$, has a spin in the x-direction given by the number $-\frac{\hbar}{2}$ in the appropriate units where the '$-$' indicates that this spin is in the opposite direction. (Cf. Figure 6 (a) in the previous handout.)

NOTE: Rather ironically, perhaps, we are not really interested in the *values* of the spin in this or that direction, but the *state* of the electron that corresponds to the electron having such a spin value. As such, we won't really talk about the spin *values* any more, concentrating instead on the corresponding spin *states* of the electron.

Another important thing that we get from the formalism is that the x-spin states and the y-spin states are related. In particular, we can see that the x-spin states are *superpositions* (or *linear combinations*) of the y-spin states, i.e.

$$|\uparrow_x\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_y\rangle + |\downarrow_y\rangle\Big) \quad \text{and} \quad |\downarrow_x\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_y\rangle - |\downarrow_y\rangle\Big),$$

and, similarly, the y-spin states are superpositions of the x-spin states, i.e.

$$|\uparrow_y\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_x\rangle + |\downarrow_x\rangle\Big) \quad \text{and} \quad |\downarrow_y\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_x\rangle - |\downarrow_x\rangle\Big),$$

where the factors of $1/\sqrt{2}$ in these expressions guarantee that the vectors are normalised. As such, by Postulate 1, these superpositions are also spin states of the system. In particular, an electron which is in an x-spin eigenstate (say up-x-spin) is in a y-spin state represented by a superposition of the y-spin eigenstates.

Lastly, we notice that the dynamics given in Postulate 3 tells us that if we don't make any measurements, then the [temporal] evolution of the state of the system will be described by the Schrödinger equation. As far as we are concerned, this means that the evolution of the system will be continuous, causal and deterministic. Or, more basically, if the system is in a superposition of eigenstates and no measurements are being made, it will stay in a superposition!

Making measurements — Postulates 4 and 5

However, when we make a measurement, things start to get a bit weird. And, it is the remaining two postulates of QM (i.e. 4 and 5) which are needed to account for this. The key point to bear in mind in QM is that when we make a measurement, we *only* observe outcomes where the system is in an eigenstate of the operator corresponding to the measurement. This is, perhaps, the crux of all the weirdness in QM, we have systems in states which are superpositions of eigenstates, but we *never* observe superpositions, we just observe the post-measurement eigenstate of the system. So, to explain what is happening with these postulates let's try and see what the formalism tells us about our earlier set of Stern-Gerlach experiments.

We only observe eigenstates — Figure 3.7

This is, pretty much what is being illustrated in Figure 3.7 (a). Whatever the x-spin state of the electrons entering the SGX-box, we will only ever get up-x-spin or down-x-spin electrons coming out. In particular, given that we are considering an SGX-box, which makes measurements corresponding to the operator $\hat{S}_x$ discussed above, an electron entering 'in' to the apparatus could be in

- a *particular* eigenstate of $\hat{S}_x$. That is, it could be in the state $|\uparrow_x\rangle$ *or* the state $|\downarrow_x\rangle$. In this case, as described by Postulate 4, the electron will stay in this eigenstate and leave the SGX-box by the appropriate hole.
- a *superposition* of the eigenstates $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$ of $\hat{S}_x$. For example, it could be in the state $|\uparrow_y\rangle$ which, as we saw above, is such a superposition as far as x-spin states are concerned. In this case, as described by Postulate 4, although the electron will leave the SGX-box by *one* of

the holes, we can only assess the probability that this electron will leave the SGX-box through the up-x-spin or down-x-spin hole. Indeed, if the electron leaves in the up-x-spin (down-x-spin) state, using Postulate 5, we say that the superposition that initially described the x-spin state of this electron has *collapsed* into the up-x-spin (down-x-spin) state.

And, of course, a similar account holds for what is happening due to the SGY-box in Figure 3.7 (b).

Spin measurements are repeatable — Figure 3.8

Notice that this first point means that, once an electron has been found to be in a particular eigenstate, say we have used an SGX-box to determine that it is in the up-x-spin state, then (provided there is no 'tampering', see below) a second measurement of its x-spin will agree with this measurement — see Figure 8 in the previous handout.

The 'statistical independence' of x and y-spins — Figure 3.9

Now, using an SGY-box to perform a y-spin measurement on an up-x-spin electron, we know from above that performing this experiment on any given electron we will find that

- the experiment will yield an electron in either an up-y-spin state or a down-y-spin state as these are the only two possible outcomes of such an experiment. As such, we invoke postulate 5 since in this measurement the superposition will *collapse* into one of these two available states, i.e. as a result of the measurement an electron in the up-x-spin state, a superposition of up-y-spin and down-y-spin states, collapses into either the up-y-spin or the down-y-spin state.

- we can see that the coefficients of the y-spin states on the right-hand-side of the expression for $|\uparrow_x\rangle$ tell us, using Born's rule (postulate 4), that the probability of obtaining one or the other of these outcomes is a half.[4]

This is why, when we measure the y-spin of many up-x-spin electrons (as illustrated in Figure 3.9) the SGY-box tells us that half of them are in an up-y-spin state and the other half are in the down-y-spin state.

Why SGY-boxes 'disturb' x-spin states — Figure 3.10

Now, we need to be clear about how a *superposition* of states, like the one discussed above, behaves. In particular, we want to distinguish superpositions from *mixtures* of different states and this is, pretty much, the point of Figure 3.10. So, just to clarify in case you haven't already realised, a superposition of the up-x-spin and down-x-spin eigenstates does *not* represent a situation where we have a set of electrons, some of which are in the former state and the rest of which are in the latter state. The superposition *is* the x-spin state of the electrons, but we only ever *observe* electrons in one of these x-spin eigenstates.

So, to see what is happening here, we note that the up-y-spin electrons leaving the SGY-box are in a state which is a superposition of the x-spin eigenstates $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$. Thus, by discovering that an electron is now in an up-y-spin state, it is no longer in an 'observable' x-spin state since it is in a superposition as far as these latter states are concerned. As such, if we now try to measure the x-spin of a large number of electrons we will find that, as above,

- the experiment will yield an electron in either an up-x-spin state or a down-x-spin state as these are the only two possible outcomes of such an experiment. As such, we invoke postulate 5 since in this measurement the superposition will *collapse* into one of these two available states, i.e. s a result of the measurement an electron in the up-y-spin state, a superposition of up-x-spin and down-x-spin states, collapses either in to the up-x-spin state or the down-x-spin state.

[4]Since, in this case, the probability of a given outcome is just the square of the appropriate coefficient in the linear combination of eigenstates that represents the superposition. See footnote 3.

- we can see that the coefficients of the x-spin states on the right-hand-side of the expression for $|\uparrow_y\rangle$ tell us, using Born's rule (postulate 4), that the probability of obtaining one or the other of these outcomes is a half.

This is why, when we now measure the x-spin of many up-y-spin electrons (as illustrated in Figure 3.10) the SGX-box tells us that half of them are in an up-x-spin state and the other half are in the down-x-spin state. That is, passing the electrons through the SGY-box 'disturbed' their x-spin state.

Why 'measurement' is so important — Figure 3.11

To round this discussion of our earlier Stern-Gerlach experiments off, we look at the final experiment. Clearly, given our earlier discussion, it should be clear why we get the results illustrated in Figure 3.11 (a), the fact that we 'count' the electrons after they leave the SGY-box means that we have ascertained which of the two y-spin eigenstates they are in. As such, when the electrons enter into the second SGX-box, they are in a superposition of the x-spin states giving the result that we would expect from our discussion of Figure 3.10.

However, what is happening in Figure 3.11 (b)? Well, although the electrons pass through the SGY-box, we do *not* use this device to make any measurement of their y-spin. As such, the superposition of y-spin states given by the up-x-spin state of the electrons which are incident on the SGY-box does *not* collapse since, by Postulate 5, collapse *only* occurs when a measurement is made. Consequently, when the electrons reach the second SGX-box they are still in the up-x-spin state and the outcome of *this* measurement, that they are all indeed in such a state, is what we observe.

Clearly, this is very weird, and accounting for the fact that it is *observing* the result of passing electrons through a measuring apparatus is what makes an interpretation of QM so difficult to find. So difficult, in fact, that many of the interpretations of QM that we will consider in the coming lectures will seem very bizarre. However, now that we have seen the formalism that physicists use, let's have a look at what has been the most enduring interpretation of it.

5 The Copenhagen Interpretation of QM

The Copenhagen interpretation is largely due to Bohr, whose model of the atom we considered in Lecture 2. This interpretation is, at least *prima facie*, the orthodox interpretation of QM amongst physicists. I say *prima facie* since what is standardly taught in undergraduate QM courses pays a great deal of lip service to what Bohr proposed without actually giving any clear account of what Bohr said. This assertion is justified by the fact that Bohr's writing are almost impenetrable and the position preached by physicists is a remarkably simplistic synopsis of what he may or may not have been trying to say.

Bohr's position is usually taken to be operationalist, instrumentalist and positivist, although a closer inspection of his work seems to show that his position is not quite so clear cut. Indeed, in this section we shall try and develop a more charitable reading of Bohr's subtle position, although the inherent vagueness of his writings won't let us get very far!

Essentially, Bohr's Copenhagen interpretation requires three things:

1. At the most fundamental level, quantum phenomena are inherently *indeterministic*.

2. It is impossible for us to give an event-by-event *causal* representation of quantum phenomena within a continuous space-time arena. (In particular, building on Heisenberg's uncertainty results, Bohr accepted that the transition of an atomic system from one state to another during an interaction could be *discontinuous*.)

3. A notion of *complementarity*. This is the most impenetrable aspect of Bohr's interpretation and so all we can do is give examples of the use of the term:

    - Examples of complementarity: the *wave and particle* aspects of a quantum system are complementary, as our its *position and momentum* and its *spin in the x and y-directions*.

- In classical physics, a system has both a definite position and a definite momentum. But, in QM, although both of these *complementary* aspects of the system are necessary for its full characterisation, it is impossible to describe the system simultaneously in terms of both of these complementary features.[5]
- '*Complementarity*: any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena.' [Bohr, 1934][6]
- '... atomic phenomena under different experimental conditions must be termed complementary in the sense that each is well defined and that together they exhaust all definable knowledge about the objects concerned. The quantum-mechanical formalism ... gives ... an exhaustive complementary account of a very large domain of experience.' [Bohr, 1958]
- The wave-particle dualism results from the complementary nature of the wave and particle descriptions of quantum phenomena. These descriptions are mutually exclusive and jointly exhaustive.[7]

Indeed, going deeper into Bohr's writings on complementarity we find that this concept starts to get quite 'mystical'.

Further reading: N. Bohr, *Atomic Theory and the Description of Nature* (CUP, 1934) and *Atomic Theory and Human Knowledge* (Wiley, 1958).

Bohr collaborated closely with Heisenberg when developing the Copenhagen interpretation. Heisenberg's take on the Copenhagen interpretation runs as follows:

- The concepts and language of classical physics, while limited by the uncertainty principle, are essential for the description of experimental results and cannot be improved upon. The probabilities in QM play an ineliminable role due to the uncertainty relations, but these probabilities are of a *fundamentally different type* to those encountered in classical physics.
- There is complementarity. But, Heisenberg's examples are different to Bohr's and he thinks that it is the 'space-time' and 'deterministic' descriptions of atomic events that are complementary.
- Reality becomes definite only when an experimental observation is made. So, prior to experiment, the probabilities give a *complete* description of what is happening. Thus, from the completeness of the probability function, it follows that electrons (say) do *not* have any particular position (say) when they are not being observed.

which, although it seems to be saying much the same sort of thing as Bohr, it doesn't seem to be quite so vague.

Further reading: W. Heisenberg, *Physics and Philosophy: the Revolution in Modern Science* (Harper and Row, 1962).

6 The perceived consequences of QM amongst philosophers and physicists

The Copenhagen interpretation was largely accepted by physicists as the best account of what was going on. There was, of course, some disagreement at the time and, in particular, as we shall see in a few weeks, Einstein put up stiff resistance to Bohr's claims. But, in the debate between Bohr and Einstein, Bohr was perceived to be the victor by the scientific community and the Copenhagen interpretation became the orthodoxy. So, bearing in mind what we have seen above, this is a quick summary of what the orthodox reading of the Copenhagen interpretation says:

[5] So, we can never give a full description of a quantum system?
[6] But, if this is right, how can a description of *both* features be the only way of characterising the system correctly?
[7] How can mutually exclusive descriptions both be true of a system?

1. We can find no deeper theory that will explain what is going on, we just have to accept that the quantum world is mysterious.

2. In the quantum world there is no fact of the matter until a measurement is made.

3. For quantum systems it is impossible to know the values of two complementary properties at the same time.[8]

4. The quantum world is fundamentally indeterministic.

Or, more philosophically, we could say that:

1. QM is **instrumentalist**: We can only use it to make predictions, it does not tell us what there is or exactly what is happening.

2. QM is **positivist**: We only know the truth or falsity of propositions when we can assess their truth or falsity by measurement.

3. QM is **operationalist**: The only quantities that we can ascribe values to our those that we can measure in a given experimental set-up.

4. QM is **indeterministic**: We cannot predict the outcome of a measurement in advance.

Needless to say, if this is how you are taught QM, it is no wonder that you never question it. The prevailing opinion nowadays amongst physicists and philosophers who think seriously about QM is that the Copenhagen interpretation managed to stifle conceptual innovation in this area of physics for a very long time. It is only now that alternative interpretations are being seriously considered and these are what we shall start looking at in the next lecture.

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[8]Technically, i.e. in the quantum mechanical formalism, *complementary* properties are those that are represented by *non-commuting* operators. For example, the operators $\hat{S}_x$ and $\hat{S}_y$ which we used in our analysis of the x and y-spin Stern-Gerlach experiments.