432018 PHILOSOPHY OF PHYSICS (Spring 2002)

Lecture 8: Interpretations which invoke incompleteness

Preliminary reading: Albert [1], Ch. 7 (pp. 134-79).

If the state vectors that describe quantum systems are *incomplete* as descriptions of the system, what is missing? Well, at the moment we can't be sure, but these missing pieces of information are generically referred to as 'hidden variables'. Now, in the last lecture we saw that there are constraints on what these hidden variables can be and, in particular, we found that they must be *contextual* in order to avoid the consequences of the Kochen-Specker theorem. Also, whatever they are, we want the theory to tally with what we expect from the first four postulates of our quantum mechanical formalism. In this lecture we will consider one of the earliest hidden variable theories, namely, Bohm's.

1 The way of incompleteness — Bohm's theory

Bohm's theory rests on the simple assumption that there are particles (pretty much in the classical sense of the term) and that, at all times, all of these particles have a definite position. At first sight this may seem like an amazing assumption given the weirdness that we have encountered in our attempts to understand what is happening in QM. After all, if we just have particles then what's the wavefunction doing? Well, in Bohm's theory, the wavefunction describes a real physical thing, called a 'pilot wave', that 'moves' particles in much the same way as water waves 'move' a cork! That is, the wavefunction gives us good information about what we expect from experiments since it is causally responsible for the motions of the particles in question.

More formally, Bohm accepts that the Schrödinger equation describes the evolution of the wavefunction at all times. But, his theory rests on a clever substitution that turns this equation into another, classical looking, equation. The upshot of this is that given the initial position of a particle we can calculate the position and momentum at a later time by using the information that we have about the 'forces'[1] acting on the particle. Now, it starts to get a bit more complicated: the forces that we have to consider here fall into two groups:

- the 'normal forces' that we need when we use the Schrödinger equation to calculate the wavefunction, and
- a so-called 'quantum force' for the system which can be found using the wavefunction.

and, needless to say, it is the 'quantum force' that gives rise to the quantum phenomena that we are expecting.

So, let's see what Bohm's strategy is:

- There are particles and they have a definite position at all times, and taking into account the classical and quantum forces acting on them we can derive their positions and momenta at all later times. As such, Bohm's theory is both causal and deterministic.
- In particular, the quantum force is calculated using the wavefunction which in turn is found by solving Schrödinger's equation which in turn contains terms which relate to the classical forces. That is, the quantum force includes information about the classical forces that are acting on the particle due to the interactions it undergoes with other systems, like measuring apparatus.
- The contextuality imposed on QM by the Kochen-Specker theorem is accounted for in Bohm's theory by the fact that the quantum force depends on the interactions that the particle has to partake in. Specifically, this means that the quantum force and the subsequent motion of the particle will depend on what measurement is being made.

[1] I use the term 'force' here and throughout since it is probably more familiar to everyone. To be precise however, there are no *forces* as such in the theory, just *potentials*.

- The probabilities which we expect from the quantum mechanical formalism arise by considering a large number of particles that have unknown *initial* positions. As such, in a manner which is familiar to physicists from Statistical Mechanics, we pick a distribution function to account for our *ignorance* of their initial positions. Indeed, picking the most 'natural' distribution, the particles behave in a way which is compatible with the probabilities we expect from the formalism of QM. In particular, these probabilities are now *epistemic*.

At this point, it should be clear that the quantum force is going to be the basis of Bohm's explanation of quantum weirdness. So, to make it clear what's happening let's look at some examples:

The Stern-Gerlach experiments

An account of how Bohm's theory accounts for the results of our earlier Stern-Gerlach experiments can be found in Albert [1], pp. 145-55.

The double-slit experiment

An account of how Bohm's theory accounts for our initial electron double-slit experiment (see Section 1.3 of the Lecture 3 handout) can be found in Holland [2], pp. 173-83. But, as this latter account is technical, let's just look at the highlights here:

- The quantum potential associated with the double-slit experiment is shown in Figures 1 and 2.
- Basically, the 'force' this potential exerts on an electron is proportional to how 'steep' the potential is at the electron's location. As such, we can see that where the potential is:
  - **'flat'**, the electron's will not experience much of a force and so they will tend to follow a straight path.
  - **'steep'**, the electron's will experience a force that makes them follow a curved path away from that point.

  The trajectories of some particles that start (at different positions) near the holes is shown in Figure 3.
- As the electron's tend to accumulate in regions where the potential is 'flattest', we can see that we will observe 'peaks' in the number of electrons detected as illustrated in Figure 4.

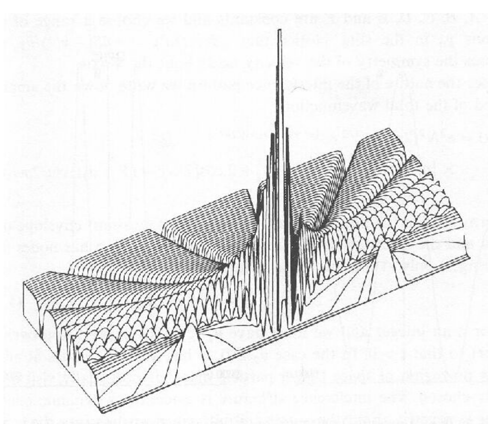

Figure 1: *The quantum potential for the double-slit experiment with the holes (located at the little 'bumps' either side of the central peak) in the foreground looking off towards the 'backstop'. (Taken from [2], p. 180.)*

And, this is what we found in Lecture 3. Also, note that if we added a light source to detect which hole the electron went through, we would change the 'normal forces' acting in the system and hence the wavefunction and hence the quantum potential. As such, any change in the experimental set-up, no matter how far away it is from the electrons we are using, changes the quantum potential and, as such, the results of the experiment. It is this that makes Bohm's theory non-local.

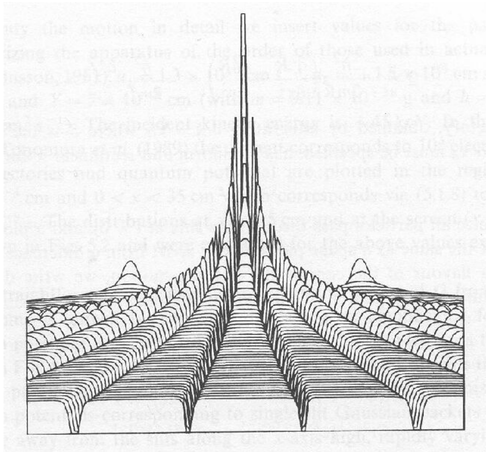

Figure 2: *The same quantum potential as we saw in Figure 1, but now viewed from the 'backstop'. (Taken from [2], p. 180.)*

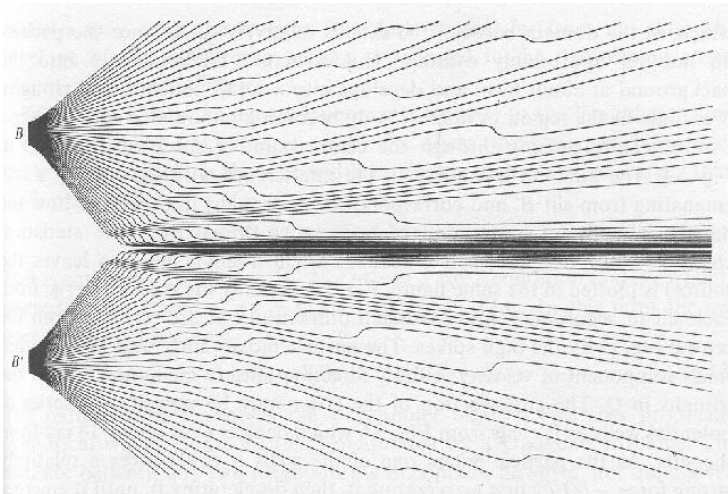

Figure 3: *If an electron starts from an initial position that is in the vicinity of the holes ('B's on the left-hand-side of the picture), its trajectory would look like one of the trajectories illustrated here. Notice that the electrons are congregating in regions where the quantum potential (seen in Figures 1 and 2) is 'flat'. (Taken from [2], p. 181.)*

Problems with Bohm's theory

But, of course, there are **problems** with this theory:

- The nice view of particle trajectories in real space being guided by the appropriate forces only works when we have one particle. Indeed, as we saw with de Broglie's matter waves, in the many particle case the trajectories are of a point (representing the entire system of particles) in a higher dimensional configuration space.[2]

- Although the quantum force acts on the particle, the particles don't act back on it. This makes a lot of people uncomfortable as it is a force which violates the 'action-reaction principle'. Furthermore, this force has no 'sources' and so where exactly does it originate?

- Even though the probabilities have been introduced in a reasonable way, there is no guarantee that the chosen distribution is the 'right' one. As such, the reason why we get the correct

[2]Indeed, when we are just looking at one particle, Albert's pictures are OK. But, they can be misleading (see his 'Bohmian' discussion of the EPR-experiment, [1] pp. 155-64), since the moment we have two particles, the trajectories of the 'two particle system' are traced out in a six-dimensional [configuration] space. Although, having said this, there is a natural mapping from the trajectories in configuration space to real space trajectories — it's just that the 'nice' physics is not happening in our three-dimensional world like it is with theories like Classical Mechanics and Electromagnetism.

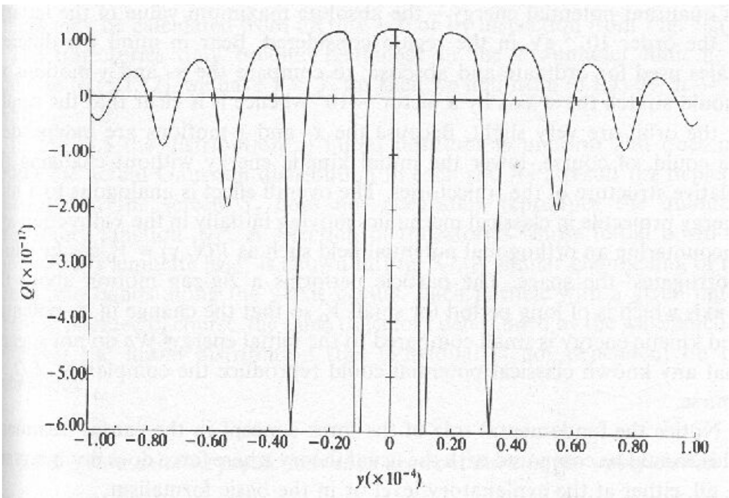

Figure 4: *The quantum potential at the 'backstop' . As electrons tend to accumulate where the quantum potential is 'flattest' we will see 'peaks' in the number of electrons detected in certain regions. These 'peaks' are what we saw in Figure 3 (c) from the Lecture 3 handout. (Taken from [2], p. 181.)*

statistics is still not fully justified — why don't other probability distributions work?[3]

- The quantum force behaves in an extremely non-local way. As we shall see in the next lecture, non-locality seems to be part-and-parcel of QM and so the quantum force has to be this way if Bohm's theory is to agree with QM. But, as we shall see, there is good reason to be suspicious of a causal mechanism, i.e. a *force*, which is non-local.

- The properties we measure, due to their contextuality as represented by the non-local behaviour of the quantum force, are properties that are associated with the 'particle plus quantum force' system. That is, the particle *itself* doesn't have any properties it just acts *as if* it has those properties due to its interaction with the quantum force.[4] So, for example, it would appear that in Bohm's theory an electron doesn't *have* spin in the sense that we normally understand when we say 'electron's have spin'.[5]

References

[1] D. Z. Albert, *Quantum Mechanics and Experience* (Harvard University Press, 1994).

[2] P. E. Holland, *The Quantum Theory of Motion* (CUP, 1993).

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[3] Although, there is a similar problem which arises in Statistical Mechanics.

[4] But, recall that the quantum force acting on a particle depends on its position, and as such, position is a property *of* the particle.

[5] What about properties like mass? This isn't 'really' a quantum property since it doesn't have an operator and we don't 'measure' it in the 'usual' quantum way. So, is this a contextual or non-contextual property of an electron? What about charge? Exactly, how do we draw the line between contextual and non-contextual properties? And, once we have drawn it, what consequences does it have for our physical and metaphysical views of properties?