Discrete Mathematics

MA210

Exercises 10

(1) Let C be the linear code of length n with check matrix

$$H = [\underbrace{1 \ 1 \ 1 \dots \ 1}_{n}].$$

Show that C is the parity check code (defined in lectures).

- (2) Let C be the d-repetition code of length n. Show that C is a linear code.
- (3) (a) Let C be the linear code with check matrix

$$H = \left[\begin{array}{rrrrr} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

Determine the length n of C, the dimension k of C, the minimum distance d of C. (We then refer to C as an [n, k, d]-code.)

(b) The following words were received:

Decide which of the above are codewords, and correct those which are not codewords, assuming that only one error has been made.

(4) Let C be the linear code with check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

If the word 110110 is received, and at most one error has been made, what was the intended codeword?

(5) (a) Let C be a code of length n. Suppose that C is 1-error-correcting. Prove that

$$|C| \le \frac{2^n}{n+1}.$$

(b) Show there is no 1-error-correcting code of length 5 with |C| = 6.

You must justify the answers to all problems!

These exercises are to be handed in **before 16.55pm on May 6, 2009**.

Lent 2009