Errata (May 12, 2006; July 22, 2009) for

HARD-TO-SOLVE BIMATRIX GAMES

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The second-to-last paragraph on page 410 should read:

It is easy to see that the shortest path lengths are obtained as follows: If d is divisible by four, that is, $d/2$ is even, then the shortest path length occurs for missing label $d/2$, and is given by $L(d,d/2) = 2a_{d/4} - 2$ according to Theorem 8(c). If $d/2$ is odd, then the shortest path length occurs for missing label $3d/2$, where $L(d,3d/2) = L(d,3d/2+1) = 2b_{(d/2+1)/2}$ by Theorem 8(b) and (d). When $d/2$ is even, the path when dropping label $3d/2$ is only two steps longer than when dropping label $d/2$ since then $L(d,3d/2) = b_{d/4} + b_{d/4+1} = b_{d/4} + a_{d/4} + c_{d/4} = 2a_{d/4}$. Therefore, the shortest path results essentially when dropping label $3d/2$.

On page 412, the statement of Lemma 11 is incorrect. Its assumption “even if it knows E” has to be changed to “that does not have any other information about E”. The corrected Lemma and its proof are as below; the first paragraph of the old proof is irrelevant. An explanation why this correction is needed follows.

LEMMA 11: *Consider a $d \times 2d$ game where a pair of supports defines a Nash equilibrium if and only both supports have size d, and player 2's support belongs to the set E, a set of d-sized subsets of $\{1,\ldots,2d\}$. Randomly permute the $2d$ pure strategies of player 2. Then a support enumeration algorithm that does not have any other information about E has to test an expected number of*

$$(19) \quad \frac{\binom{2d}{d} - |E|}{|E|+1} + 1$$

supports before finding an equilibrium support.

PROOF: By assumption, the algorithm does not gain any information from negative trials. Consequently, any order of testing d-sized supports is equally good on average. A standard argument (Motwani and Raghavan (1995), p. 10) then shows that the expected number of support guesses until an equilibrium is found is given by (19), as claimed. *Q.E.D.*

To show why the old statement is incorrect, consider the game $\Gamma(2,4)$ in Lemma 10. The unpermuted bit strings v that represent equilibrium strategies of player 2 are of the form $11t$ with $t \in \{1100, 0011, 1001\}$. Suppose t is permuted in any of the 4! possible ways with equal probability. According to (19), the expected number of guesses is 1.75, which can also be explicitly computed as $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{3}{5} \cdot 2 + \frac{1}{2} \cdot \frac{2}{5} \cdot \frac{3}{4} \cdot 3 + \frac{1}{2} \cdot \frac{2}{5} \cdot \frac{1}{4} \cdot 4$, with the integers showing the number of trials.

We claim that the sequence of guesses 1100, 1010, 0110 for t results in a smaller expected number of guesses, namely $5/3$, computed as $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot \frac{1}{3} \cdot 3$, where $\frac{2}{3} = \frac{1}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{3}{4}$. Clearly, the first guess 1100 is correct with probability $1/2$. If not, 1100 is a permutation of one of the three non-equilibrium strings (a) 0110, (b) 1010, or (c) 0101. Then changing the guess 1100 to the second guess 1010 corresponds to one of the following changes of these non-equilibrium strings:

(a) 0110 changed to 0011 or (*) 1010 (by moving the first "1"), or
0110 changed to 1100 or (*) 0101 (by moving the second "1");
case (a) occurs with probability $1/3$ and has a success rate of $1/2$.

(b) 1010 changed to (*) 0110, or 0011 (by moving the first "1"), or
1010 changed to 1100 or 1001 (by moving the second "1");
case (b) occurs with probability $1/3$ and has a success rate of $3/4$;

(c) is analogous to (b), with probability $1/3$ and success rate $3/4$.

Overall, the second guess, needed with probability $1/2$, has the claimed success rate $\frac{2}{3} = \frac{1}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{3}{4}$.

The second guess 1010 is unsucessful in the cases marked (*) above. Then it is easy to see that the third guess 0110 corresponds to an equilibrium string. This third trial is needed with probability $\frac{1}{2} \cdot \frac{1}{3}$.