Example for the KM structure theorem

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Consider an $m \times n$ bimatrix game (A,B), with payoff matrix A for the row player (player I) and B for the column player (player II). Our example is the 3×3 game

$$A = \begin{bmatrix} 0 & 9 & 0 \\ 3 & 0 & 3 \\ -9 & 12 & 15 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 3 & -6 \\ 6 & 0 & 9 \\ 6 & 3 & 0 \end{bmatrix}.$$

Consider now the row averages a for player I and column averages b for player II:

$$a = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}, \qquad b = [4 \quad 2 \quad 1].$$

We write $(A,B) = (\tilde{A},\tilde{B}) \oplus (a,b)$, i.e.

$$A = \begin{bmatrix} -3 & 6 & -3 \\ 1 & -2 & 1 \\ -15 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 6 & 6 & 6 \end{bmatrix}, \qquad B = \begin{bmatrix} -4 & 1 & -7 \\ 2 & -2 & 8 \\ 2 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 1 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}.$$

We consider the equilibrium (x,y) of (A,B) with

$$x = \begin{bmatrix} 0 \\ 2/3 \\ 1/3 \end{bmatrix}, \qquad y = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}, \quad \text{so that} \quad Ay = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}, \qquad B^\top x = \begin{bmatrix} 6 \\ 1 \\ 6 \end{bmatrix},$$

and thus

$$Ay + x = \begin{bmatrix} 0 \\ 3\frac{2}{3} \\ 3\frac{1}{3} \end{bmatrix}, \qquad B^\top x + y = \begin{bmatrix} 6\frac{1}{2} \\ 1 \\ 6\frac{1}{2} \end{bmatrix}.$$

The vector $(Ay + x, B^\top y + x)$ is in general written as $G\sigma + \sigma$. It is the sum, for each pure strategy, of the expected payoff for that strategy and the equilibrium probability.

Note the use of the best response condition!

Unrelated: Lemke-Howson diagrams for this game: