Repeated Games
Played With Adaptive Automata

A Progress Report on the Reciprocity Game

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Cooperation via reciprocity

Reciprocity is "exchange for mutual benefit"

Examples of continuing relationships:

- marriage, partnership, lender-borrower [repo market]
- symbiosis/mutualism [bee-flower, coral-amoeba]
- social insects [ants, bees, termites]

E.O. Wilson (Sociobiology) conjectures that a successful species evolves toward intra-species cooperation

- hypothesis — an evolutionary stable strategy (ESS) reciprocates cooperation
- implication — outcome of subgame perfect equilibrium (SPE) is cooperation (EES $\Rightarrow$ proper equilibrium $\Rightarrow$ SPE)

The Repeated Prisoner's Dilemma

The PD stage-game: 2 players, each cooperates or defects
— where defect dominates cooperate

Repeated PD: payoff is average of stage-game payoffs

- finite repetition: Nash equilibrium outcome is always-defect, but game perturbed by tit-for-tat has unique sequential equilibrium — cooperation until near the end of the game
  - small probability one must use tit-for-tat $\Rightarrow$ imitate tit-for-tat $\Rightarrow$ other cooperates $\Rightarrow$ both cooperate
- infinite repetition: subgame-perfect equilibria (SPE) yield all outcomes — this is the "folk theorem"
  - still true with bounded-recall strategies [Sabourian] implemented by a simple automaton
  - ! but, for recall = 1 period, iterative elimination of dominated strategies shows that the only stable outcome is cooperation [Aumann]

Source of the Difference

Two kinds of finite automata

- Simple automaton specifies an action for each state
  - e.g. a state is recalled portion of any history
- Adaptive automaton repeats the action when the same state recurs along a path of play
  - strategy develops along each path
    — like a computer programmed sequentially
  - Adaptive automata correspond to evolutionary processes

For both kinds of automata, each path eventually cycles, so a player's payoff is his average stage-game payoff in the cycle.

Henceforth assume strategies are adaptive automata

- in this case, stable outcome ⇒ SPE outcome
  as computed by backward induction from cycle closures

The Repeated Reciprocity Game

Players alternate, each giving the other a gift, or not
— this is the repeated PD game with alternating actions

- action C: **Cooperation**: player i giving a gift incurs cost c_i and yields benefit b_j to the other player j
- action D: **Defect**: not giving a gift yields 0 to both players

Assume $b_i > c_i > 0$ so that reciprocal gift-giving is mutually beneficial and efficient.
If each ratio b_i/c_i is generic then:

Theorem

There is a unique SPE outcome, hence a unique stable outcome

- this contrasts with the "folk theorem"
- ⋆ What is this unique stable outcome?

Computational results, using backward induction

For recalls 1,2,3,4 there are 16, 124, 4364, and 2,054,560 paths of max lengths 5, 10, 19, and 36

Theorem

For bounded-recall strategies with recalls ≤ 4, the SPE outcome is always-cooperate

- Note: there are also paths that start badly, then continue with always-cooperate

For the modified game based on cycles of payoffs rather than actions (a la Rubinstein-style bargaining):

Theorem

For bounded-recall strategies with payoff recalls ≤ 16, the SPE outcome is always-cooperate

- Cycles of payoffs occur earlier along paths of play, which enables solution of larger games

General results

Theorem

Always-defect is not the SPE outcome — some cooperation occurs

Theorem

When each player is always able to cooperate after other's tit-for-tat behavior, the SPE outcome is always-cooperate

- This is a quasi-theorem — we have NOT been able to prove that a player is always able to cooperate on the SPE path
- But theorem's conclusion is true if a player can increase the size of his automaton for a small cost
- In evolutionary context, some mutations increase the size of the genome (Fudenberg-Maskin)

More quasi-theorems

Theorem

In the SPE, if reply to unprovoked Defect is to Defect, then the SPE outcome is always-cooperate

- That is, if unprovoked Defects are punished then . . . .

Theorem

On a path without two unprovoked Defects in succession, the SPE continuation is always-cooperate

- That is, SPE continuation is always-cooperate
  on paths that are not too far off the cooperative path

Where do we stand?

Contrary to the "folk theorem", game theory makes a unique prediction — but we have not proved that it is always-cooperate

- Uniqueness of prediction comes from
  (1) strategies implementable by adaptive automata, and
  (2) the stronger solution concepts of SPE or stability

Is E.O. Wilson's hypothesis supported?

- It might require every-larger automata — i.e. size of genome might need to increase to enable cooperation
- Tit-for-tat requires only 2-state automaton, but it is not SPE (even for recall 1) because far off the cooperative path, one player can exploit the other's vulnerability