Repeated Games
Played With Adaptive Automata

A Progress Report on the Reciprocity Game

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Cooperation via reciprocity

Reciprocity is “exchange for mutual benefit”

Examples of continuing relationships:

- marriage, partnership, lender-borrower [repo market]
- symbiosis/mutualism [bee-flower, coral-amoeba]
- social insects [ants, bees, termites]

E.O. Wilson (Sociobiology) conjectures that a successful species evolves toward intra-species cooperation

- hypothesis — an evolutionary stable strategy (ESS) reciprocates cooperation
- implication — outcome of subgame perfect equilibrium (SPE) is cooperation (EES ⇒ proper equilibrium ⇒ SPE)
TheRepeatedPrisoner’sDilemma

The PD stage-game: 2 players, each cooperates or defects — where defect dominates cooperate

Repeated PD: payoff is average of stage-game payoffs

- **finite** repetition: Nash equilibrium outcome is always-defect, but game perturbed by tit-for-tat has unique sequential equilibrium — cooperation until near the end of the game
  - small probability one must use tit-for-tat ⇒ imitate tit-for-tat ⇒ other cooperates ⇒ both cooperate
- **infinite** repetition: subgame-perfect equilibria (SPE) yield all outcomes — this is the “folk theorem”
  - still true with bounded-recall strategies [Sabourian] implemented by a simple automaton
  - ! but, for recall = 1 period, iterative elimination of dominated strategies shows that the only stable outcome is cooperation [Aumann]
Two kinds of finite automata

- **Simple** automaton specifies an action for each state
  - e.g. a state is recalled portion of any history
- **Adaptive** automaton repeats the action when the same state recurs along a path of play
  - strategy develops along each path — like a computer programmed sequentially
  - Adaptive automata correspond to evolutionary processes

For both kinds of automata, each path eventually cycles, so a player’s payoff is his average stage-game payoff in the cycle.

Henceforth assume strategies are **adaptive automata**

- in this case, stable outcome $\Rightarrow$ SPE outcome as computed by backward induction from cycle closures
The Repeated Reciprocity Game

Players alternate, each giving the other a gift, or not — this is the repeated PD game with alternating actions

- action C: **Cooperation**: player $i$ giving a gift incurs cost $c_i$ and yields benefit $b_j$ to the other player $j$
- action D: **Defect**: not giving a gift yields 0 to both players

Assume $b_i > c_i > 0$ so that reciprocal gift-giving is mutually beneficial and efficient.

If each ratio $b_i/c_i$ is generic then:

**Theorem**

*There is a unique SPE outcome, hence a unique stable outcome*

- this contrasts with the “folk theorem”
  - What is this unique stable outcome?
Computational results, using backward induction

For recalls 1, 2, 3, 4 there are 16, 124, 4364, and 2,054,560 paths of max lengths 5, 10, 19, and 36

**Theorem**

*For bounded-recall strategies with recalls \( \leq 4\), the SPE outcome is always-cooperate*

- Note: there are also paths that start badly, then continue with always-cooperate

For the modified game based on cycles of payoffs rather than actions (a la Rubinstein-style bargaining):

**Theorem**

*For bounded-recall strategies with payoff recalls \( \leq 16\), the SPE outcome is always-cooperate*

- Cycles of payoffs occur earlier along paths of play, which enables solution of larger games
General results

**Theorem**

*Always-defect is not the SPE outcome — some cooperation occurs*

**Theorem**

*When each player is always able to cooperate after other’s tit-for-tat behavior, the SPE outcome is always-cooperate*

- This is a quasi-theorem — we have **NOT** been able to prove that a player is always able to cooperate on the SPE path
- But theorem’s conclusion is true if a player can increase the *size* of his automaton for a small cost
- In evolutionary context, some mutations increase the *size* of the genome (Fudenberg-Maskin)
**Theorem**

*In the SPE, if reply to unprovoked Defect is to Defect, then the SPE outcome is always-cooperate*

- That is, if unprovoked Defects are punished then . . . .

**Theorem**

*On a path without two unprovoked Defects in succession, the SPE continuation is always-cooperate*

- That is, SPE continuation is always-cooperate on paths that are not too far off the cooperative path
Where do we stand?

Contrary to the “folk theorem”, game theory makes a unique prediction — but we have not proved that it is always-cooperate

- Uniqueness of prediction comes from
  1. strategies implementable by adaptive automata, and
  2. the stronger solution concepts of SPE or stability

Is E.O. Wilson’s hypothesis supported?

- It might require ever-larger automata — i.e. size of genome might need to increase to enable cooperation
- Tit-for-tat requires only 2-state automaton, but it is not SPE (even for recall 1) because far off the cooperative path, one player can exploit the other’s vulnerability