What Is The Game? Thinking and Over-Thinking Strategically

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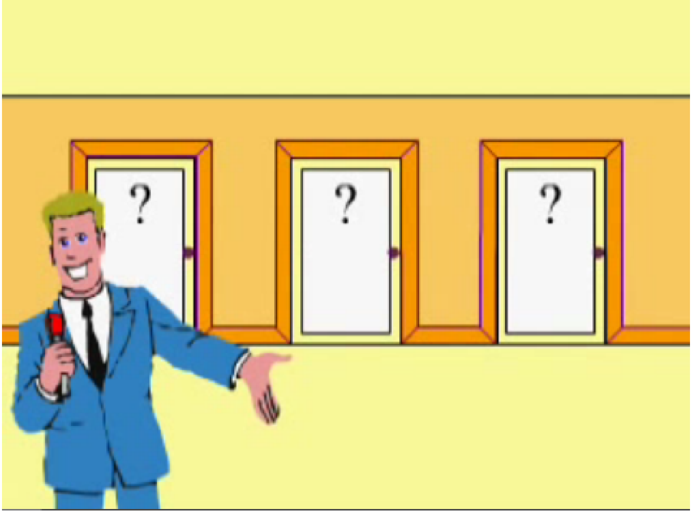
[Click on references for URLs]

Overview

Advanced game theory models are not taken seriously in practice.

- Two example uses of game theory (kept simple!)
- What is game theory good for? Shubik's flavours of game theory (from management consulting)
- What we need:
  - real (not artificial) problems that demonstrate the challenges of game theory
  - descriptive tools about agents' view of situations
- Example: Duopoly with demand inertia. Combines search for optimal play and evolutionary game theory

The Monty Hall Problem



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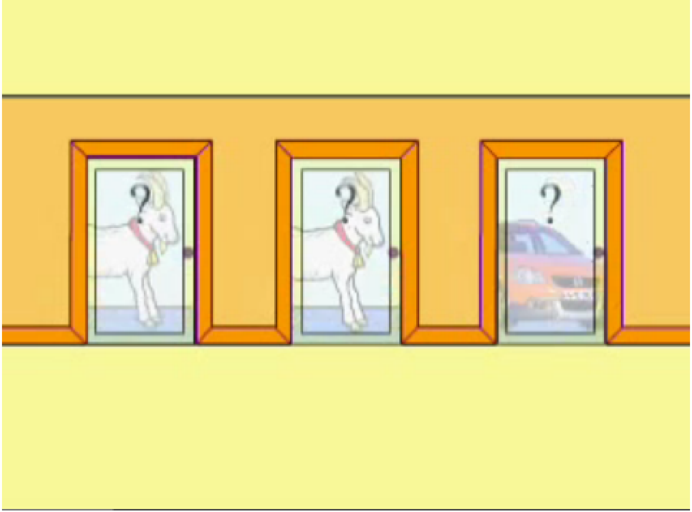

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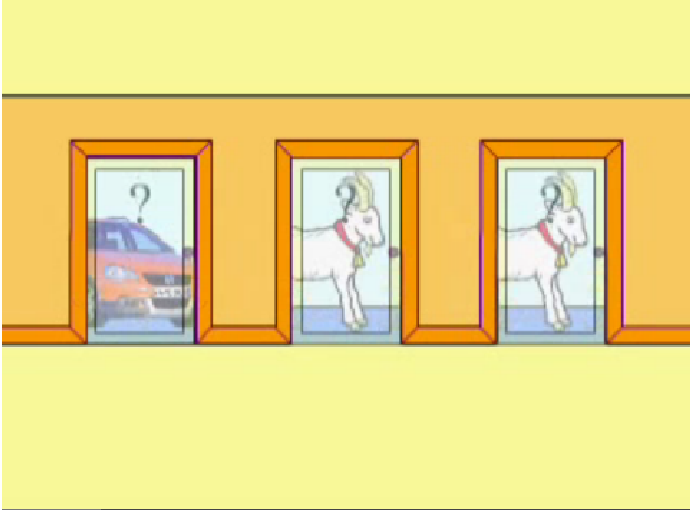

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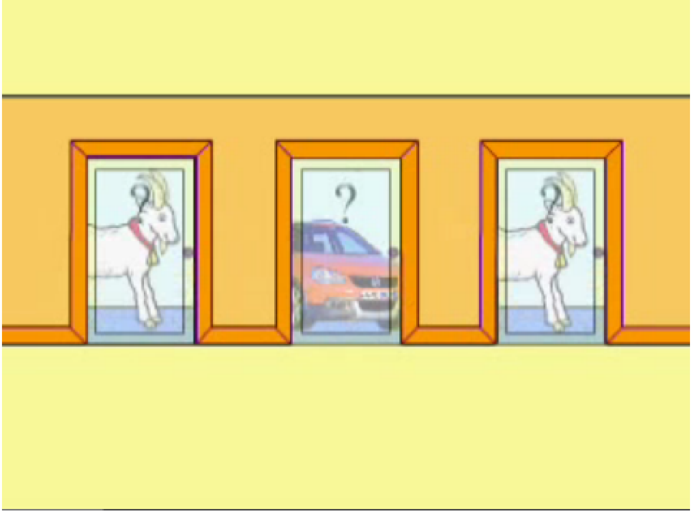

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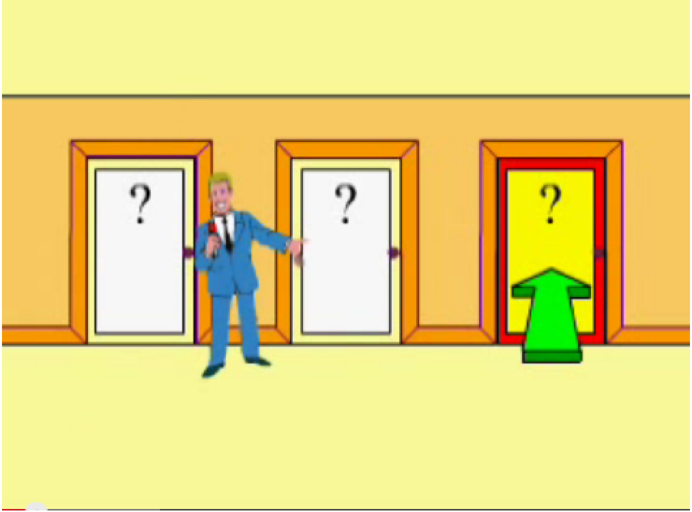

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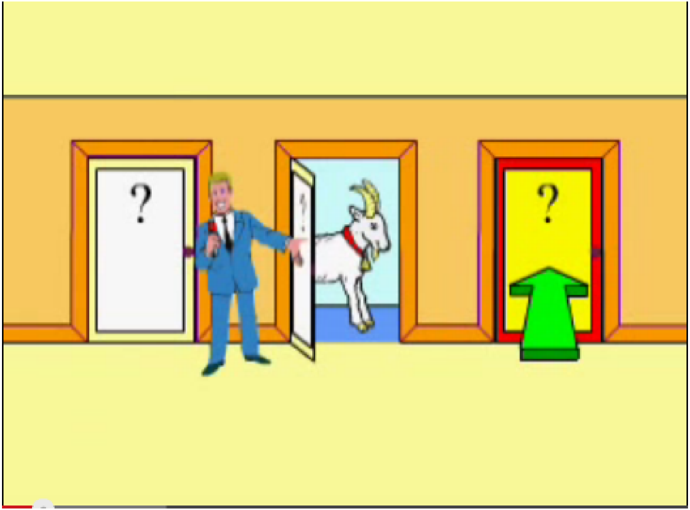


The Monty Hall Problem

The standard solution

The candidate has with probability $\mathbf{1/3}$ chosen the correct door, which does not change when the show host, Monty Hall, opens another door and shows the goat.

So when offered to switch to the remaining door, the candidate **should switch**, thereby raising the probability that he will get the car to $\mathbf{2/3}$.

A variant, now a game

Consider now the following game-theoretic variant:

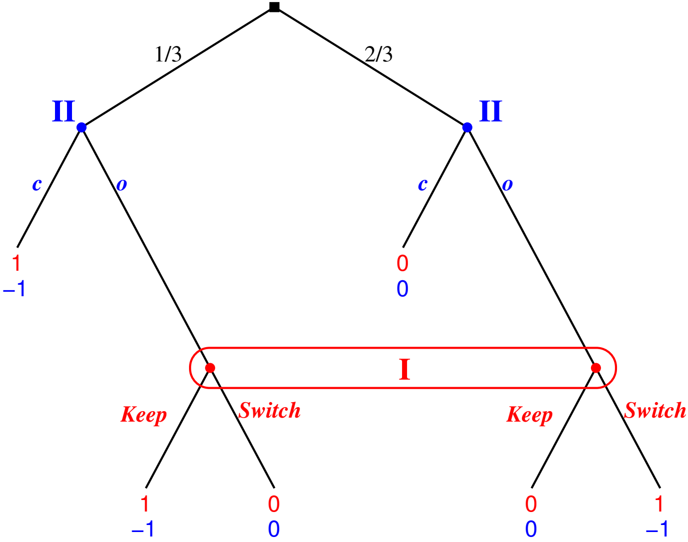
- The candidate, player I, chooses one of the doors.
- Monty Hall, player II, knows if that door has the car or not.

A variant, now a game

Consider now the following game-theoretic variant:

- The candidate, player I, chooses one of the doors.
- Monty Hall, player II, knows if that door has the car or not.
- player II now has the **option** to open another door (move ***o***) that has a goat behind it, or keep the doors closed (move ***c***).
- If player II chose ***o***, then player I now can choose to Keep his original choice or Switch to the other closed door.
- player I wants to maximize the probability of winning the car, and player II wants to minimize it.
- These rules are known to the players.

Game tree



Strategic form and optimal strategies

I \ II	*cc*	*co*	*oc*	*oo*
Keep	1/3	1/3	1/3	1/3
Switch	1/3	1	0	2/3

Only one maxmin strategy, ***Keep***, of player I .

Strategic form and optimal strategies

maxmin strategy

	I \ II	*cc*	*co*	*oc*	*oo*
1	*Keep*	1/3	1/3	1/3	1/3
0	*Switch*	1/3	1	0	2/3

Only one maxmin strategy, ***Keep***, of player I .

Four extreme minmax strategies of player II .

Strategic form and optimal strategies

minmax strategy

II / I	1 *cc*	0 *co*	0 *oc*	0 *oo*
Keep	1/3	1/3	1/3	1/3
Switch	1/3	1	0	2/3

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minmax strategy

	0	0	1	0
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Strategic form and optimal strategies

I \ II	*cc*	*co*	*oc*	*oo*
minmax strategy	**0**	**0**	**1/2**	**1/2**
Keep	**1/3**	**1/3**	**1/3**	**1/3**
Switch	**1/3**	**1**	**0**	**2/3**

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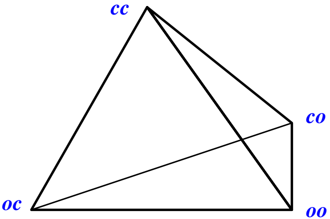
Optimal strategies in the simplex



minmax strategy	0	1/3	2/3	0
	0	0	1/2	1/2
	0	0	1	0
	1	0	0	0
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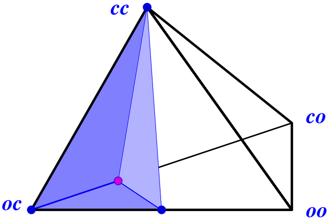

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	0	0	1/2	1/2
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	1	0	0	0
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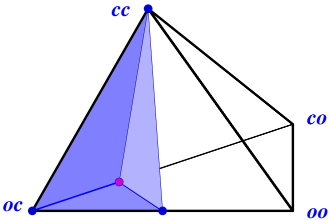

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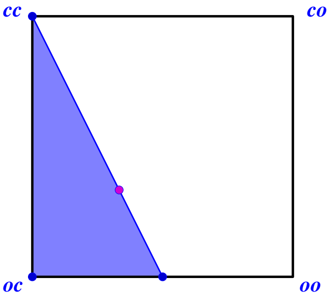

← minmax behaviour strategies

Lessons from the Monty Hall game

- Rules of the game matter and are often ambiguous
- Game theory helps analyze a game with **clear** rules (use separate analysis for each possible rule set)
- People may be suspicious of being tricked

Conventions matter in negotiations

Example: House sale

Rubinstein's bargaining model: alternating rejections and counter-offers. **Discount factor** can be interpreted as probability of **negotiation breakdown** after rejection.
(Stationary SPNE approximates Nash bargaining solution.)

- Maintaining **trust** between parties is paramount.

[A. Rubinstein (1982), Perfect equilibrium in a bargaining model. *Econometrica* 50, 97–109]

[B. von Stengel (2022), *Game Theory Basics*, Cambridge Univ. Press]

Keep protocols simple

Example: House sale

Multiple buyers willing to pay more than the asking price.

Sealed bids: Would you run a **second-price auction?**

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Do you understand VCG (Vickrey-Clark-Groves)?

It is meant to induce **truthful bidding** but needs to be understood, trusted (and constructed in the first place). Usually none applies.

Simple mechanisms beat complicated truthful mechanisms.

Where is Game Theory used?

Martin Shubik: 3 flavours of game theory

- **High-church** game theory: mathematical concepts such as
  - Nash equilibrium
  - core, Shapley value
  - often l'art pour l'art
- **Low-church** game theory: Applications e.g. to oligopoly theory
- **Conversational** game theory: Management consulting

[M. Shubik (1987), What is an application and and when is theory a waste of time? *Management Science* 33, 1511–1522]

[M. Shubik (2002), Game theory and Operations Research: Some musings 50 years later. *Operations Research* 50, 192–196]

Low-church game theory

- Advice tool for technical advice on
  - weapons evaluation (zero-sum games)
  - search and evasion games, inspection games
  - tactical advertising and marketing models
- not nearly as useful as, say, linear programming
- often contrived
  - nobody believes a model, except the person who made it
  - everybody believes observations, except the person who made them
- Is game theory too unrealistic? Too idealized?

Conversational Game Theory

Uses concepts from high-church game theory:

- zero-sum game, non-zero-sum game
- Prisoner's Dilemma

Shubik rejects Schelling (*Strategy of Conflict*), but later appreciates his willingness to **change the rules of the game**.

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Use in **management consulting:**

- clarify local near-term strategic goals
- changing modes of thought
- **paramount:** know your business

Economic engineering

Market design and practical **auction design** are success stories of game theory

- requires detailed understanding of the underlying constraints

[R. Wilson (2002), Architecture of power markets. *Econometrica* 70, 1299–1340]

Computational complexity

It is useful to understand exponential growth of

- number of game tree nodes in terms of **depth**
  - Shubik: “Few long-range plans can afford the luxury of working out more than a few alternative paths”
- number of pure strategies in terms of **tree nodes** (remedied with behaviour strategies)
- possible number of **mixed Nash equilibria**
- number of possible lexicographic belief systems, partitions, . . .
- steps of computing one Nash equilibrium (seems rare).

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But: to be understandable/solvable, most games are **small**.

Common knowledge

Knowledge about actions (and about knowledge) is a useful concept.

Common knowledge can make a difference

- Emperor's new clothes [Pinker (2011), Why indirect speech?]
- "overlook" embarrassing accidental common knowledge

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Claim: n-level thinking (for $n \gg 1$) is too contrived (centipede game, traveller's dilemma).

Claim: **Common priors** are too contrived.

Common assumptions about behaviour are probably ubiquitous but rarely spelled out.

What is needed for a more realistic game theory

Not: Artificial models of bounded rationality (automata,...).

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- models of how agents see the game (typically partial, from their perspective, and evolving), and matching solution concepts
- in line with **perception as controlled prediction**
  [Anil Seth (2021), *Being You: A New Science of Consciousness.*]

Example from Selten's research

[C. Keser, A. Gaudeul (2016), Foreword: Special issue in honor of Reinhard Selten's 85th birthday. *German Economic Review* 17, 277–283]

Model: **Duopoly with demand inertia**

- analysed theoretically (**perfect** equilibrium)

[R. Selten (1965), Game-theoretic analysis of an oligopolic model with buyers' interia. [German] *Zeitsch. gesammte Staatswiss.* 21, 301–304]

- experimentally with subjects and submitted programmed strategies

[C. Keser (1993), Some results of experimental duopoly markets with demand inertia. *Journal of Industrial Economics* 41, 133–151]

[1992 PhD thesis: Springer Lecture Notes Econ. Math. Systems 391]

Duopoly with demand inertia

Total demand potential **400** split as $D_1 + D_2$ between two producers with costs $c_1 = 57$ and $c_2 = 71$.

Producer i chooses **price** p_i and **sells** $D_i - p_i$ units, gets **profit** $(p_i - c_i)(D_i - p_i)$.

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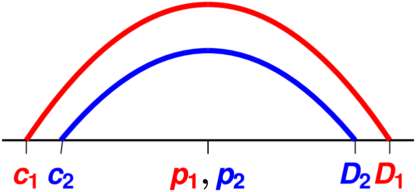

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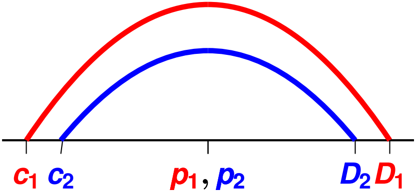
Optimal **myopic** price $p_i = (c_i + D_i)/2$. **Example:**

$D_1 = 207$, $D_2 = 193$, $p_1 = p_2 = 132$, profits 75^2, 61^2.



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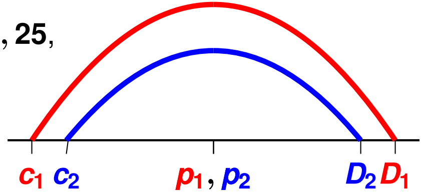
Played over 25 periods $t = 1, \ldots, 25$,

$$D_1^1 = D_1^1 = 200$$

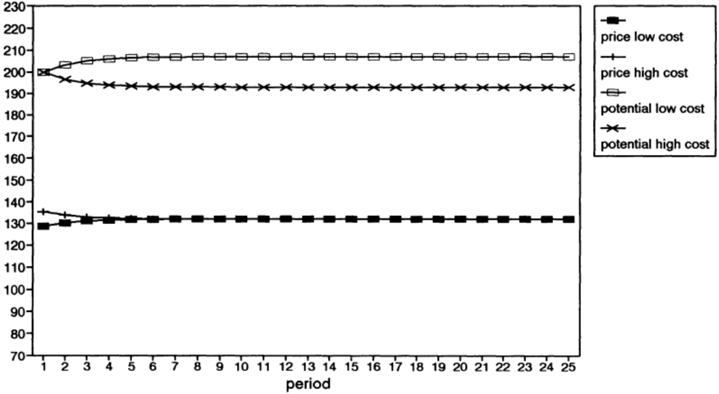
$$D_1^{t+1} = D_1^t + (p_2^t - p_1^t)/2$$

$$D_2^{t+1} = D_2^t + (p_1^t - p_2^t)/2$$



Cooperative solution

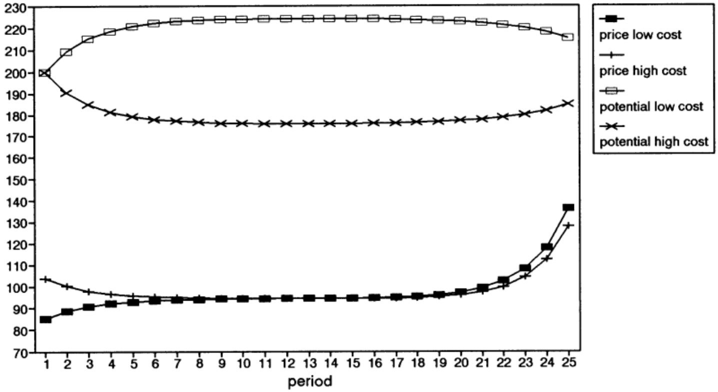
If both producers always choose myopic duopoly price:



Total profits over 25 periods about **156**k, **109**k

Subgame perfect equilibrium

Via parameterized backward induction:

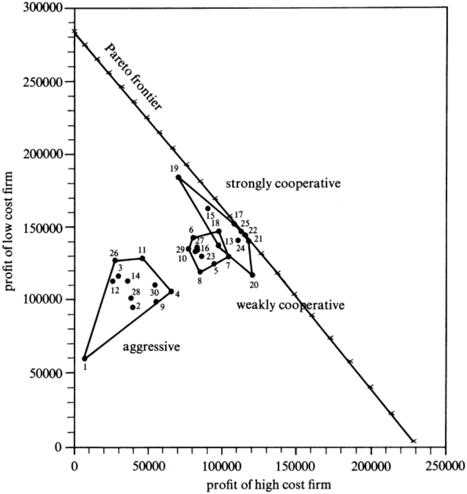


Total profits about **137**k, **61**k

Strategy experiments

Teams submitted flowcharts for low-cost and high-cost firms.
Competition rounds 1 (45 entries) and 2 (34) with feedback.

self-play results:



Lessons from a participant's perspective

Profits were totalled against all other teams (including own type)

- **Very important for doing well:** understanding the game
  - focus on demand potential, not price
  - smaller price strongly increases future profits
  - exploit "suckers"
  - avoid wild swings

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- Strategies **react** (typically to last price) but have **no model of the opponent**
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- "Optimization" of parameters typically against self-play.

Evolutionary game extension

L \ H	*a*	*b*	*c*	*d*
A	**100** / 152	86 / **180**	98 / 157	99 / 154
B	**110** / 74	66 / 170	47 / **178**	75 / 130
C	102 / **155**	**103** / 160	**103** / 156	**103** / 157
D	**105** / 154	**105** / 158	**105** / 155	104 / **159**

Evolutionary game extension

	L \ H	0.05 *a*	0.03 *b*	0.58 *c*	0.34 *d*
0.02	*A*	**100** 152	86 **180**	98 157	99 154
0.01	*B*	**110** 74	66 170	47 **178**	75 130
0.67	*C*	102 **155**	**103** 160	**103** 156	**103** 157
0.30	*D*	**105** 154	**105** 158	**105** 155	104 **159**

equilibrium payoffs:

103

156

The unplayed round 3 of this strategy experiment

The evolutionary stage was not planned, and a round 3 might have led to changed strategies for greater evolutionary success:

- mixed equilibrium probabilities are determined by **opponent** payoffs – also as population fractions?
- would players realistically program such strategies?
- this would be a special two-stage game:
  - stage 1 (over 25 periods): determine your own payoff row (against unknown opponents)
  - stage 2: swim in the evolutionary pool

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- this would be a special two-stage game:
  - stage 1 (over 25 periods): determine your own payoff row (against unknown opponents)
  - stage 2: swim in the evolutionary pool
- two research questions:
  - (theory) how to play optimally, or **learn** to do so
  - (experiment) how people **try** to play optimally

Why I like this problem

Stage 1: play to determine your payoff row
Stage 2: automatically run evolutionary dynamics

Why I like this problem

Stage 1: play to determine your payoff row
Stage 2: automatically run evolutionary dynamics

- A neat high-church game theory model (many variations possible)
- demand inertia (or similar) is somewhat realistic
- stages 1 and 2 combine rational / optimization approach with automatic / blind evolutionary game theory
- we need to **combine** rationality with automatism / fitness
- as observed in reality, this may reflect local rationality with interaction protocols that have evolved to work.