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Pricing of contingent claims in a two-dimensional model with random dividends*

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We study a model of a financial market in which two risky assets are paying dividends with rates changing their initial values to other constant ones when certain events occur. Such events are associated with the first times at which the value processes of issuing firms, modeled by geometric Brownian motions, fall to some prescribed levels. The asset price dynamics are described by exponential diffusion processes with random drift rates and independent driving Brownian motions. We derive closed form expressions for rational values of European contingent claims, under full and partial information.

1 Introduction

In the present paper, we study a first passage time model for two dividend paying assets with dividend rates changing their initial values to other constant ones, during the allowed infinite time horizon. The times of change of the dividend rates are assumed to be the first times at which the firm values hit some given lower constant barriers. Such a model corresponds to a financial market in which the fall of one of the firm values leads to a change of the dividend rate

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not of the asset issued by the same firm only, but of the other ones too. For instance, such a situation may happen in a model with one parent company having several branches (or firms), when a financial trouble of one of the firms makes an influence on the dividend policy of all other ones as well. Note that some other models with random dividends were earlier considered in the literature (see, e.g. Geske [3]), where the possibility of significance of stochastic dividend effect on the rational values of contingent claims was emphasized. We introduce a dividend switching model for asset prices that reflects certain contagion effect between low ratings of several firms, which appears to be new for the related literature, to the best of our knowledge. Other first passage time contagion models were considered within the credit risk framework, for example, in Zhou [15], Giesecke [4], Overbeck and Schmidt [10], Valužis [14] (see also Bielecki and Rutkowski [1; Chapter X] or Schönbucher [13; Chapter X] for further references).

The purpose of the present paper is to derive closed form expressions for rational values of European contingent claims in the model described above. Suppose that the evolution of firm values (or some related indices) is modeled by geometric Brownian motions, and the risk-neutral dynamics of the underlying asset prices are described by exponential diffusion processes having the following structure. Assume that the drift rates of the latter processes are changing their initial values to other constant ones at the first times when the former processes fall to some prescribed levels, on the infinite time interval. For simplicity of exposition, we restrict our consideration to the two-dimensional case and assume that the firm values and the underlying asset price processes are driven by the same Brownian motions, which are supposed to be independent of each other. The rational values of the claims are expressed through the transition density of the joint marginal distribution of a geometric Brownian motion and its running minimum, as well as through the density of its first passage time on a constant level. The results of the paper can naturally be extended to the case of a model with several underlying assets, the price processes of which are driven by independent Brownian motions with random drift rates. A generalization of the model to the case of correlated driving Brownian motions would lead to more complicated and less explicit formulas (see, e.g. Iyengar [6], He et al. [5] or Patras [11]).

The paper is organized as follows. In Section 2, we introduce a two-dimensional model with two firms issuing two underlying risky assets having the dividend structure described above. In Section 3, we derive expressions for rational prices of European contingent claims with respect to the filtration generated by both firm value processes (full information). In Section 4, we present expressions for the same contingent claims with respect to the filtration generated by the value of one of the firms only (partial information). In Section 5, we illustrate our results on the rational pricing of European exchange options allowing its holders to exchange one asset for another (see, e.g. Margrabe [9]), in the model described above, under full information. In that case, the expressions for rational prices can be simplified, and thus become more amenable for simulations. The main results of the paper are stated in Propositions 3.1 and 4.1.

2 The model

In this section, we introduce a first passage model for two firms issuing dividend paying assets.

2.1 The dynamics of firm values and asset prices

We consider a probability space $(\Omega, \mathcal{G}, P)$ with two *independent* standard Brownian motions $W^i = (W^i_t)_{t \geq 0}$, $i = 1, 2$. Let the processes $X^i = (X^i_t)_{t \geq 0}$, $i = 1, 2$, be given by:

$$X^i_t = x_i \exp\left(\left(\eta_i - \frac{\theta_i^2}{2}\right) t + \theta_i W^i_t\right) \tag{2.1}$$

where η_i, $\theta_i > 0$ and $x_i > 0$ are some constants, for every $i = 1, 2$. The processes X^i, $i = 1, 2$, describe the evolution of values of two firms. Following the structural approach in credit risk models, let us define the random variables τ_i, $i = 1, 2$, by:

$$\tau_i = \inf\{t \geq 0 \,|\, X^i_t \leq b_i\} \tag{2.2}$$

where $b_i > 0$, $i = 1, 2$, are some given constant levels. We will sometimes use the notation $\tau_i(x_i)$ to emphasize the dependence of τ_i on the starting value x_i, for every $i = 1, 2$. Note that, by construction, τ_i, $i = 1, 2$, are stopping times with respect to the natural filtration $\mathcal{G}_t = \sigma(X^1_u, X^2_u \,|\, 0 \leq u \leq t)$, $t \geq 0$, of the process (X^1, X^2).

Let the processes $S^i = (S^i_t)_{t \geq 0}$, $i = 1, 2$, be given by:

$$S^i_t = s_i \exp\left(\left(r - \frac{\sigma_i^2}{2} - \delta_{i,0}\right) t - (\delta_{i,1} - \delta_{i,0})\,(t - \tau_1)^+ - (\delta_{i,2} - \delta_{i,0})\,(t - \tau_2)^+ + \sigma_i W^i_t\right) \tag{2.3}$$

where $(t - \tau_i)^+ = \max\{t - \tau_i, 0\}$, and σ_i, $\delta_{i,k}$, s_i are some strictly positive constants, for every $i = 1, 2$ and $k = 0, 1, 2$. The existence of such a pair of processes (S^1, S^2) can be easily deduced from the classical diffusion model with constant dividend rates, by means of standard change-of-measure arguments. The processes S^i, $i = 1, 2$, describe the risk-neutral dynamics of the prices of dividend paying assets issued by the two firms. Here, $r \geq 0$ is the interest rate of a riskless banking account. Observe that it follows from (2.1) and (2.3) that the processes S^i, $i = 1, 2$, admit the representation:

$$S_t^i = s_i \left(\frac{X_t^i}{x_i} \right)^{\alpha_i} \exp\left(rt + \beta_{i,0} t + \gamma_{i,1} (t - \tau_1)^+ + \gamma_{i,2} (t - \tau_2)^+ \right) \tag{2.4}$$

where $\alpha_i = \sigma_i / \theta_i$, $\beta_{i,0} = \sigma_i \theta_i / 2 - \sigma_i \eta_i / \theta_i - \sigma_i^2 / 2 - \delta_{i,0}$ and $\gamma_{i,k} = \delta_{i,0} - \delta_{i,k}$, $i, k = 1, 2$.

At the random times τ_i, $i = 1, 2$, at which the value processes of the two firms hit some prescribed barriers, the dividend rates of the underlying assets change their initial values to other constant ones. In more details, for every $i = 1, 2$ fixed, the i-th asset pays dividends at the rate $\delta_{i,0}$ until the time $\tau_1 \wedge \tau_2 = \min\{\tau_1, \tau_2\}$ at which the first event occurs and the dividend rate is changed to $\delta_{i,\ell}$, where $\ell = 1$ if $\tau_1 \wedge \tau_2 = \tau_1$, and $\ell = 2$ if $\tau_1 \wedge \tau_2 = \tau_2$. Then, the i-th asset pays dividends with the rate $\delta_{i,\ell}$ until the time $\tau_1 \vee \tau_2 = \max\{\tau_1, \tau_2\}$ at which the second event occurs and the dividend rate is changed to $\delta_{i,3} = \delta_{i,1} + \delta_{i,2} - \delta_{i,0}$. After both events occur, the i-th asset pays dividends with the rate $\delta_{i,3}$.

2.2 The payoffs of European contingent claims

The purpose of the present paper is to determine the rational (no-arbitrage) prices of European contingent claims with payoffs of the form $C(S_T^1, S_T^2)$, for some non-negative measurable functions $C(s_1, s_2)$, $s_i > 0$, $i = 1, 2$, and a fixed time horizon $T > 0$. Without loss of generality, we assume that the payoffs are already discounted by the banking account, that is equivalent to letting r equal to zero. The rational (or no-arbitrage) price process $V = (V_t)_{0 \le t \le T}$ of such a claim is given by:

$$V_t = E[C(S_T^1, S_T^2) \,|\, \mathcal{G}_t] \tag{2.5}$$

for any $0 \le t \le T$, where the expectation is taken with respect to the equivalent martingale measure. In the sequel, we will provide a closed form expression for the rational price of the so-called currency exchange (or Margrabe) option with the payoff $C(s_1, s_2) = (s_1 - s_2)^+$, $s_i > 0$, $i = 1, 2$.

Observe that the value in (2.5) can be decomposed as:

$$\begin{aligned} V_t = \; & E[C(S_T^1, S_T^2) \, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \\ & + \sum_{i=1}^{2} E[C(S_T^1, S_T^2) \, I(\tau_{3-i} \le T < \tau_i) \,|\, \mathcal{G}_t] \\ & + \sum_{i=1}^{2} E[C(S_T^1, S_T^2) \, I(\tau_{3-i} < \tau_i \le T) \,|\, \mathcal{G}_t] \end{aligned} \tag{2.6}$$

for all $0 \leq t \leq T$, where $I(\cdot)$ denotes the indicator function. It follows from the expression in (2.4) that the value process in (2.6) admits the representation:

$$
\begin{aligned}
V_t = {} & E[C_0(T, X_T^1, X_T^2)\, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \\
& + \sum_{i=1}^{2} E[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i})\, I(\tau_{3-i} \leq T < \tau_i) \,|\, \mathcal{G}_t] \\
& + \sum_{i=1}^{2} E[C_{2,i}(T, \tau_{3-i}, \tau_i, X_T^i, X_T^{3-i})\, I(\tau_{3-i} < \tau_i \leq T) \,|\, \mathcal{G}_t]
\end{aligned} \tag{2.7}
$$

for any $0 \leq t \leq T$. Here, according to the expression in (2.4), for each $T > 0$ and the starting values s_i, x_i, $i = 1, 2$, we have $C_0(T, y, z) = C(D_0^1(T, y), D_0^2(T, z))$, where we define $D_0^i(T, x) = s_i (x/x_i)^{\alpha_i} e^{\beta_{i,0} T}$ with $\alpha_i = \sigma_i/\theta_i$ and $\beta_{i,0} = \sigma_i \theta_i/2 - \sigma_i \eta_i/\theta_i - \sigma_i^2/2 - \delta_{i,0}$, $i = 1, 2$. We also have $C_{1,i}(T, v, y, z) = C(D_i^i(T, v, y), D_i^{3-i}(T, v, z))$, where we define $D_i^\ell(T, v, x) = s_\ell (x/x_\ell)^{\alpha_\ell} e^{\beta_{\ell,0} T + \gamma_{\ell,3-i}(T-v)}$ with $\gamma_{\ell,i} = \delta_{\ell,0} - \delta_{\ell,i}$, $i, \ell = 1, 2$. Moreover, we have $C_{2,i}(T, v, u, y, z) = C(D_3^i(T, v, u, y),\ D_3^{3-i}(T, v, u, z))$, where we define $D_3^\ell(T, v, u, x) = s_\ell (x/x_\ell)^{\alpha_\ell} e^{\beta_{\ell,0} T + \gamma_{\ell,3-i}(T-v) + \gamma_{\ell,i}(T-u)}$, $i, \ell = 1, 2$.

Furthermore, we shall also determine the rational prices of the European contingent claims under the assumption that the information available from the market is generated by one of the firm values only. This corresponds to a situation where small investors trading in the market cannot observe the values (or related indices) of other firms. In that case, the rational price processes $V^j = (V_t^j)_{0 \leq t \leq T}$, $j = 1, 2$, of the claims are given by:

$$
\begin{aligned}
V_t^j = {} & E[C_0(T, X_T^1, X_T^2)\, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t^j] \\
& + \sum_{i=1}^{2} E[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i})\, I(\tau_{3-i} \leq T < \tau_i) \,|\, \mathcal{G}_t^j] \\
& + \sum_{i=1}^{2} E[C_{2,i}(T, \tau_{3-i}, \tau_i, X_T^i, X_T^{3-i})\, I(\tau_{3-i} < \tau_i \leq T) \,|\, \mathcal{G}_t^j]
\end{aligned} \tag{2.8}
$$

for any $0 \leq t \leq T$. Here, $\mathcal{G}_t^j = \sigma(X_u^j \,|\, 0 \leq u \leq t)$, $t \geq 0$, is the natural filtration of the process X^j, for every $j = 1, 2$.

2.3 The minimum process and some distribution laws

Let us introduce the running minimum process $M^i = (M_t^i)_{t \geq 0}$ associated with the process X^i and defined by:

$$
M_t^i = \min_{0 \leq u \leq t} X_u^i \wedge m_i \tag{2.9}
$$

for any $x_i \geq m_i > 0$ fixed and $i = 1, 2$. It is known (see, e.g. [12; Chapter III, Section 3], [7; Appendix E] or [2; Part II, Section 2]) that the transition density g_i of the Markov process (X^i, M^i) defined by:

$$P_{x_i,m_i}(X_t^i \in dx, M_t^i \in dm) = g_i(x_i, m_i; t, x, m)\, dx\, dm \tag{2.10}$$

admits the representation:

$$g_i(x_i, m_i; t, x, m) = \frac{2}{\theta_i^3 \sqrt{2\pi t^3}} \frac{\ln(m^2/(x_i x))}{xm} \exp\left(-\frac{\ln^2(m^2/(x_i x))}{2\theta_i^2 t} + \frac{\rho_i}{\theta_i} \ln(x/x_i) - \frac{\rho_i^2 t}{2}\right) \tag{2.11}$$

for all $t > 0$ and $x \geq m$ with $x_i \geq m_i \geq m > 0$, and equals zero otherwise. Here, P_{x_i,m_i} denotes the probability under the assumption that (X^i, M^i) starts at (x_i, m_i), and we set $\rho_i = \eta_i/\theta_i - \theta_i/2$. Although the expression for g_i in (2.11) does not depend on m_i explicitly, we shall keep this notation in order to further use the (strong) Markov property of the couple (X^i, M^i), for every $i = 1, 2$.

It is also known that the density h_i of the hitting time τ_i in (2.2) defined by:

$$P_{x_i,m_i}(\tau_i \in dt) = h_i(x_i; t)\, dt \tag{2.12}$$

admits the representation:

$$h_i(x_i; t) = \frac{\ln(x_i/b_i)}{\theta_i \sqrt{2\pi t^3}} \exp\left(-\frac{(\ln(x_i/b_i) + \rho_i \theta_i t)^2}{2\theta_i^2 t}\right) \tag{2.13}$$

for all $t > 0$ and $x_i \geq m_i > b_i > 0$.

3 The case of full information

In this section, we compute the three conditional expectations of the expression in (2.7).

3.1 The first term

Let us begin by computing the first term in (2.7). For this, applying the Markov property of the process (X^1, M^1, X^2, M^2), we get:

$$\begin{aligned} &E_{x_1,m_1,x_2,m_2}[C_0(T, X_T^1, X_T^2)\, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \qquad (3.1) \\ &= I(t < \tau_1 \wedge \tau_2)\, E_{x_1,m_1,x_2,m_2}[C_0(T, X_T^1, X_T^2)\, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \\ &= I(t < \tau_1 \wedge \tau_2)\, E_{X_t^1,M_t^1,X_t^2,M_t^2}[C_0(T', X_{T'}^1, X_{T'}^2)\, I(T' < \tau_1' \wedge \tau_2')] \end{aligned}$$

where we set $T' = T - t$ and $\tau_i' = \tau_i(X_t^i) \equiv \tau_i(x_i) - t$, for each $0 \le t \le T$. Here, E_{x_1,m_1,x_2,m_2} denotes the expectation under the assumption that the process (X^1, M^1, X^2, M^2) starts at (x_1, m_1, x_2, m_2) with $x_i \ge m_i > b_i > 0$, for every $i = 1, 2$. Then, using the fact that the event $\{\tau_i > t\}$ can be represented in the form $\{M_t^i > b_i\}$, we have:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_0(T', X_{T'}^1, X_{T'}^2)\, I(T' < \tau_1' \wedge \tau_2')] \\
&= E_{x_1,m_1,x_2,m_2}[C_0(T', X_{T'}^1, X_{T'}^2)\, I(M_{T'}^1 > b_1, M_{T'}^2 > b_2)]
\end{aligned} \tag{3.2}
$$

where $\tau_i' = \tau_i(x_i)$, for every $i = 1, 2$. Hence, we obtain from (3.1) and (3.2) that:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_0(T, X_T^1, X_T^2)\, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \\
&= I(t < \tau_1 \wedge \tau_2) \int_{b_1}^{\infty} \int_{b_1}^{\infty} \int_{b_2}^{\infty} \int_{b_2}^{\infty} C_0(T-t, x_1', x_2') \prod_{\ell=1}^{2} g_\ell(X_t^\ell, M_t^\ell; T-t, x_\ell', m_\ell')\, dx_\ell'\, dm_\ell'
\end{aligned} \tag{3.3}
$$

where the functions g_i, $i = 1, 2$, are given in (2.11) above.

3.2 The second term

Let us continue with computing the second term in (2.7). For this, applying the Markov property of the process (X^1, M^1, X^2, M^2), we get:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i})\, I(\tau_{3-i} \le T < \tau_i) \,|\, \mathcal{G}_t] \\
&= E_{x_1,m_1,x_2,m_2}[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i})\, I(\tau_{3-i} \le t < T < \tau_i) \,|\, \mathcal{G}_t] \\
&\quad + E_{x_1,m_1,x_2,m_2}[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i})\, I(t < \tau_{3-i} \le T < \tau_i) \,|\, \mathcal{G}_t] \\
&= I(\tau_{3-i} \le t < \tau_i)\, E_{X_t^1, M_t^1, X_t^2, M_t^2}[C_{1,i}^0(T', X_{T'}^i, X_{T'}^{3-i})\, I(T' < \tau_i')] \\
&\quad + I(t < \tau_1 \wedge \tau_2)\, E_{X_t^1, M_t^1, X_t^2, M_t^2}[C_{1,i}^1(T', \tau_{3-i}', X_{T'}^i, X_{T'}^{3-i})\, I(\tau_{3-i}' \le T' < \tau_i')]
\end{aligned} \tag{3.4}
$$

for all $0 \le t \le T$. Here, for each $T' \equiv T - t > 0$ and the starting values s_i, x_i, we have $C_{1,i}^0(T', y, z) = C(D_i^i(T-t, 0, y), D_i^{3-i}(T-t, 0, z))$ and $C_{1,i}^1(T', v, y, z) = C(D_i^i(T-t, v-t, y), D_i^{3-i}(T-t, v-t, z))$, where the functions $D_i^\ell(T, v, x)$, $i, \ell = 1, 2$, are defined above.

We then continue with computing every term in (3.4) separately. Firstly, we see that:

$$
E_{x_1,m_1,x_2,m_2}[C_{1,i}^0(T', X_{T'}^i, X_{T'}^{3-i})\, I(T' < \tau_i')] = E_{x_1,m_1,x_2,m_2}[C_{1,i}^0(T', X_{T'}^i, X_{T'}^{3-i})\, I(M_{T'}^i > b_i)] \tag{3.5}
$$

for $x_i \ge m_i > b_i > 0$ and $b_{3-i} \wedge x_{3-i} \ge m_{3-i} > 0$. Then, using the independence of (X^i, M^i)

and (X^{3-i}, M^{3-i}), we have:

$$E_{x_1,m_1,x_2,m_2}[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i}) I(\tau_{3-i} \le t < T < \tau_i) \,|\, \mathcal{G}_t] \tag{3.6}$$
$$= I(\tau_{3-i} \le t < \tau_i) \int_0^\infty \int_{b_i}^\infty \int_0^\infty \int_0^{b_{3-i}} C_{1,i}^0(T-t, x_i', x_{3-i}') \prod_{\ell=i}^{3-i} g_\ell(X_t^\ell, M_t^\ell; T-t, x_\ell', m_\ell')\, dx_\ell'\, dm_\ell'$$

where the functions g_i, $i = 1, 2$, are given in (2.11) above.

Now, applying the strong Markov property of (X^1, M^1, X^2, M^2), we have:

$$\begin{aligned} &E_{x_1,m_1,x_2,m_2}[C_{1,i}(T', \tau_{3-i}', X_{T'}^i, X_{T'}^{3-i})\, I(\tau_{3-i}' \le T' < \tau_i')] \\ &= E_{x_1,m_1,x_2,m_2}[C_{1,i}(T', \tau_{3-i}', X_{T'}^i, X_{T'}^{3-i})\, I(M_{T'}^i > b_i, \tau_{3-i}' \le T')] \\ &= E_{x_1,m_1,x_2,m_2}[\widehat{C}_{1,i}(T', \tau_{3-i}', X_{\tau_{3-i}'}^i, M_{\tau_{3-i}'}^i, X_{\tau_{3-i}'}^{3-i}, M_{\tau_{3-i}'}^{3-i})\, I(\tau_{3-i}' \le T')] \\ &= E_{x_1,m_1,x_2,m_2}[\widehat{C}_{1,i}(T', \tau_{3-i}, X_{\tau_{3-i}}^i, M_{\tau_{3-i}}^i, b_{3-i}, b_{3-i})\, I(\tau_{3-i} \le T')] \end{aligned} \tag{3.7}$$

for $x_i \ge m_i > b_i > 0$, $i = 1, 2$, where the functions $\widehat{C}_{1,i}$, $i = 1, 2$, are defined by:

$$\widehat{C}_{1,i}(T', v, x_i, m_i, x_{3-i}, m_{3-i}) = E_{x_1,m_1,x_2,m_2}[C_{1,i}^1(T', v, X_{T'-v}^i, X_{T'-v}^{3-i})\, I(M_{T'-v}^i > b_i)] \tag{3.8}$$

for $x_i \ge m_i > b_i > 0$, $b_{3-i} \wedge x_{3-i} \ge m_{3-i} > 0$ and any $0 \le v \le T'$ fixed. Thus, using the independence of τ_{3-i} and (X^i, M^i), we obtain from (3.7) that:

$$\begin{aligned} &E_{x_1,m_1,x_2,m_2}[C_{1,i}(T, \tau_{3-i}, X_T^i, X_T^{3-i})\, I(t < \tau_{3-i} \le T < \tau_i) \,|\, \mathcal{G}_t] \\ &= I(t < \tau_1 \wedge \tau_2) \int_0^{T-t} \int_0^\infty \int_{b_i}^\infty \widehat{C}_{1,i}(T-t, v, x_i', m_i', b_{3-i}, b_{3-i})\, h_{3-i}(X_t^{3-i}; v) \\ &\qquad \times g_i(X_t^i, M_t^i; v, x_i', m_i')\, dv\, dx_i'\, dm_i' \end{aligned} \tag{3.9}$$

where, by virtue of the independence of (X^i, M^i) and (X^{3-i}, M^{3-i}), it follows from (3.8) that:

$$\widehat{C}_{1,i}(T-t, v, x_i', m_i', x_{3-i}', m_{3-i}') \tag{3.10}$$
$$= \int_0^\infty \int_{b_i}^\infty \int_0^\infty \int_0^{b_{3-i}} C_{1,i}^1(T-t, v, x_i'', x_{3-i}'') \prod_{\ell=i}^{3-i} g_\ell(x_\ell', m_\ell'; T-t-v, x_\ell'', m_\ell'')\, dx_\ell''\, dm_\ell''$$

and the functions g_i and h_i, $i = 1, 2$, are given in (2.11) and (2.13) above.

3.3 The third term

Let us complete with computing the third term in (2.7). For this, applying the Markov property of the process (X^1, M^1, X^2, M^2), we get:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_{2,i}(T,\tau_{3-i},\tau_i,X_T^i,X_T^{3-i})\,I(\tau_{3-i}<\tau_i\le T)\,|\,\mathcal{G}_t] \qquad (3.11)\\
&= E_{x_1,m_1,x_2,m_2}[C_{2,i}(T,\tau_{3-i},\tau_i,X_T^i,X_T^{3-i})\,I(\tau_{3-i}<\tau_i\le t)\,|\,\mathcal{G}_t]\\
&\quad + E_{x_1,m_1,x_2,m_2}[C_{2,i}(T,\tau_{3-i},\tau_i,X_T^i,X_T^{3-i})\,I(\tau_{3-i}\le t<\tau_i\le T)\,|\,\mathcal{G}_t]\\
&\quad + E_{x_1,m_1,x_2,m_2}[C_{2,i}(T,\tau_{3-i},\tau_i,X_T^i,X_T^{3-i})\,I(t<\tau_{3-i}<\tau_i\le T)\,|\,\mathcal{G}_t]\\
&= I(\tau_{3-i}<\tau_i\le t)\,E_{X_t^1,M_t^1,X_t^2,M_t^2}[C_{2,i}^0(T',X_{T'}^i,X_{T'}^{3-i})]\\
&\quad + I(\tau_{3-i}\le t<\tau_i)\,E_{X_t^1,M_t^1,X_t^2,M_t^2}[C_{2,i}^1(T',\tau_i',X_{T'}^i,X_{T'}^{3-i})\,I(\tau_i'\le T')]\\
&\quad + I(t<\tau_1\wedge\tau_2)\,E_{X_t^1,M_t^1,X_t^2,M_t^2}[C_{2,i}^2(T',\tau_{3-i}',\tau_i',X_{T'}^i,X_{T'}^{3-i})\,I(\tau_{3-i}'<\tau_i'\le T')]
\end{aligned}
$$

for all $0 \le t \le T$. Here, for each $T' \equiv T-t > 0$ and the starting values s_i, x_i fixed, we have $C_{2,i}^0(T',y,z) = C(D_3^i(T-t,0,0,y), D_3^{3-i}(T-t,0,0,z))$, $C_{2,i}^1(T',u,y,z) = C(D_3^i(T-t,0,u-t,y), D_3^{3-i}(T-t,0,u-t,z))$ and $C_{2,i}^2(T',v,u,y,z) = C(D_3^i(T-t,v-t,u-t,y), D_3^{3-i}(T-t,v-t,u-t,z))$, where the functions $D_3^i(T,v,u,x)$, $i=1,2$, are defined above.

We now continue with computing every term in (3.11) separately. Firstly, using the independence of the processes (X^i, M^i) and (X^{3-i}, M^{3-i}), we obtain:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_{2,i}(T,\tau_{3-i},\tau_i,X_T^i,X_T^{3-i})\,I(\tau_{3-i}<\tau_i\le t)\,|\,\mathcal{G}_t] \qquad (3.12)\\
&= I(\tau_{3-i}<\tau_i\le t)\int_0^\infty\int_0^{b_i}\int_0^\infty\int_0^{b_{3-i}} C_{2,i}^0(T-t,x_i',x_{3-i}')\prod_{\ell=i}^{3-i} g_\ell(X_t^\ell,M_t^\ell;T-t,x_\ell',m_\ell')\,dx_\ell'\,dm_\ell'
\end{aligned}
$$

where the functions g_i, $i=1,2$, are given in (2.11) above.

Secondly, applying the strong Markov property of (X^1, M^1, X^2, M^2), we have:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_{2,i}^1(T',\tau_i',X_{T'}^i,X_{T'}^{3-i})\,I(\tau_i'\le T')] \qquad (3.13)\\
&= E_{x_1,m_1,x_2,m_2}[\widehat{C}_{2,i}(T',\tau_i',X_{\tau_i'}^i,M_{\tau_i'}^i,X_{\tau_i'}^{3-i},M_{\tau_i'}^{3-i})\,I(\tau_i'\le T')]\\
&= E_{x_1,m_1,x_2,m_2}[\widehat{C}_{2,i}(T',\tau_i,b_i,b_i,X_{\tau_i}^{3-i},M_{\tau_i}^{3-i})\,I(\tau_i\le T')]
\end{aligned}
$$

for $x_i \ge m_i > b_i > 0$ and $b_{3-i} \wedge x_{3-i} \ge m_{3-i} > 0$, where the functions $\widehat{C}_{2,i}$, $i=1,2$, are defined by:

$$
\widehat{C}_{2,i}(T',u,x_i,m_i,x_{3-i},m_{3-i}) = E_{x_1,m_1,x_2,m_2}[C_{2,i}^1(T',u,X_{T'-u}^i,X_{T'-u}^{3-i})] \qquad (3.14)
$$

for $b_i \wedge x_i \geq m_i > 0$, $i = 1, 2$, and any $0 \leq u \leq T'$ fixed. Thus, using the independence of τ_i and (X^{3-i}, M^{3-i}), we obtain from (3.13) that:

$$\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_{2,i}(T, \tau_i, X_T^i, X_T^{3-i})\, I(\tau_{3-i} \leq t < \tau_i \leq T) \,|\, \mathcal{G}_t] \qquad (3.15) \\
&= I(\tau_{3-i} \leq t < \tau_i) \int_0^{T-t} \int_0^{\infty} \int_0^{b_{3-i}} \widehat{C}_{2,i}(T-t, u, b_i, b_i, x'_{3-i}, m'_{3-i})\, h_i(X_t^i; u) \\
&\qquad \times g_{3-i}(X_t^{3-i}, M_t^{3-i}; u, x'_{3-i}, m'_{3-i})\, du\, dx'_{3-i}\, dm'_{3-i}
\end{aligned}$$

where, by virtue of the independence of (X^i, M^i) and (X^{3-i}, M^{3-i}), it follows from (3.14) that:

$$\begin{aligned}
&\widehat{C}_{2,i}(T-t, u, x'_i, m'_i, x'_{3-i}, m'_{3-i}) \qquad (3.16) \\
&= \int_0^{\infty} \int_0^{b_i} \int_0^{\infty} \int_0^{b_{3-i}} C^1_{2,i}(T-t, u, x''_i, x''_{3-i}) \prod_{\ell=i}^{3-i} g_\ell(x'_\ell, m'_\ell; T-t-u, x''_\ell, m''_\ell)\, dx''_\ell\, dm''_\ell
\end{aligned}$$

and the functions g_i and h_i, $i = 1, 2$, are given in (2.11) and (2.13) above.

Finally, we see that:

$$\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C^2_{2,i}(T', \tau'_{3-i}, \tau'_i, X^i_{T'}, X^{3-i}_{T'})\, I(\tau'_{3-i} < \tau'_i \leq T')] \qquad (3.17) \\
&= E_{x_1,m_1,x_2,m_2}[C^2_{2,i}(T', \tau'_{3-i}, \tau'_i, X^i_{T'}, X^{3-i}_{T'})\, I(M^i_{\tau'_{3-i}} > b_i, \tau'_i \leq T')] \\
&= E_{x_1,m_1,x_2,m_2}[\widetilde{C}_{2,i}(T', \tau'_{3-i}, X^i_{\tau'_{3-i}}, M^i_{\tau'_{3-i}}, X^{3-i}_{\tau'_{3-i}}, M^{3-i}_{\tau'_{3-i}})\, I(M^i_{\tau'_{3-i}} > b_i, \tau'_i \leq T')] \\
&= E_{x_1,m_1,x_2,m_2}[\widetilde{C}_{2,i}(T', \tau_{3-i}, X^i_{\tau_{3-i}}, M^i_{\tau_{3-i}}, b_{3-i}, b_{3-i})\, I(M^i_{\tau_{3-i}} > b_i, \tau_i \leq T')]
\end{aligned}$$

for $x_i \geq m_i > b_i > 0$, $i = 1, 2$, where the functions $\widetilde{C}_{2,i}$, $i = 1, 2$, are defined by:

$$\begin{aligned}
&\widetilde{C}_{2,i}(T', v, x_i, m_i, x_{3-i}, m_{3-i}) \qquad (3.18) \\
&= E_{x_1,m_1,x_2,m_2}[\overline{C}_{2,i}(T', v, \tau'_i, X^i_{\tau'_i}, M^i_{\tau'_i}, X^{3-i}_{\tau'_i}, M^{3-i}_{\tau'_i})\, I(\tau'_i \leq T' - v)] \\
&= E_{x_1,m_1,x_2,m_2}[\overline{C}_{2,i}(T', v, \tau_i, b_i, b_i, X^{3-i}_{\tau_i}, M^{3-i}_{\tau_i})\, I(\tau_i \leq T' - v)]
\end{aligned}$$

for $x_i \geq m_i > b_i > 0$, $b_{3-i} \wedge x_{3-i} \geq m_{3-i} > 0$ and any $0 \leq v \leq T'$ fixed, where the functions $\overline{C}_{2,i}$, $i = 1, 2$, are defined by:

$$\overline{C}_{2,i}(T', v, u, x_i, m_i, x_{3-i}, m_{3-i}) = E_{x_1,m_1,x_2,m_2}[C^2_{2,i}(T', v, u, X^i_{T'-u}, X^{3-i}_{T'-u})] \qquad (3.19)$$

for $b_i \wedge x_i \geq m_i > 0$, $i = 1, 2$, and any $0 \leq u \leq T' - v$ fixed. Hence, using again the independence of τ_{3-i} and (X^i, M^i), we obtain from (3.17) that:

$$
\begin{aligned}
&E_{x_1,m_1,x_2,m_2}[C_{2,i}(T,\tau_{3-i},\tau_i,X_T^i,X_T^{3-i})\, I(t<\tau_{3-i}<\tau_i\le T)\,|\,\mathcal{G}_t] \\
&= I(t<\tau_1\wedge\tau_2)\int_0^{T-t}\int_{b_i}^{\infty}\int_{b_i}^{\infty}\widetilde{C}_{2,i}(T-t,v,x_i',m_i',b_{3-i},b_{3-i})\,h_{3-i}(X_t^{3-i};v) \\
&\qquad\qquad \times g_i(X_t^i,M_t^i;v,x_i',m_i')\,dv\,dx_i'\,dm_i'
\end{aligned}
\tag{3.20}
$$

where, by virtue of the independence of τ_i and (X^{3-i}, M^{3-i}), it follows from (3.18) that:

$$
\begin{aligned}
&\widetilde{C}_{2,i}(T-t,v,x_i',m_i',x_{3-i}',m_{3-i}') \\
&= \int_0^{T-t-v}\int_0^{\infty}\int_0^{b_{3-i}}\overline{C}_{2,i}(T-t,v,u,b_i,b_i,x_{3-i}'',m_{3-i}'')\,h_i(x_i';u) \\
&\qquad\qquad \times g_{3-i}(x_{3-i}',m_{3-i}';u,x_{3-i}'',m_{3-i}'')\,du\,dx_{3-i}''\,dm_{3-i}''
\end{aligned}
\tag{3.21}
$$

the functions $\overline{C}_{2,i}$, $i=1,2$, admit the representation:

$$
\begin{aligned}
&\overline{C}_{2,i}(T-t,v,u,x_i',m_i',x_{3-i}',m_{3-i}') \\
&= \int_0^{\infty}\int_0^{b_i}\int_0^{\infty}\int_0^{b_{3-i}} C_{2,i}^2(T-t,v,u,x_i'',x_{3-i}'')\prod_{\ell=i}^{3-i} g_\ell(x_\ell',m_\ell';T-t-u,x_\ell'',m_\ell'')\,dx_\ell''\,dm_\ell''
\end{aligned}
\tag{3.22}
$$

and the functions g_i and h_i, $i=1,2$, are given in (2.11) and (2.13).

Therefore, summarizing the facts proved above, we are now ready to formulate the following assertion.

Proposition 3.1. *The rational price of the European contingent claim in (2.7) under full information is given by the sum of the terms in (3.3), (3.6), (3.9), (3.12), (3.15) and (3.20).*

4 The case of partial information

In this section, we describe the computation of conditional expectations in (2.8).

Let us proceed by computing the terms in (2.8). For this, let $H(t, x_j, m_j, x_{3-j}, m_{3-j})$ be a nonnegative continuous function, for any $j = 1, 2$ fixed. Then, using the independence of τ_j

and (X^{3-j}, M^{3-j}), we see that:

$$E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(t < \tau_j \wedge \tau_{3-j}) \, | \, \mathcal{G}_t^j] \tag{4.1}$$
$$= I(t < \tau_j) \, E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(M_t^{3-j} > b_{3-j}) \, | \, \mathcal{G}_t^j]$$
$$= I(t < \tau_j) \int_{b_{3-j}}^{\infty} \int_{b_{3-j}}^{\infty} H(t, X_t^j, M_t^j, x'_{3-j}, m'_{3-j}) \, g_{3-j}(x_{3-j}, m_{3-j}; t, x'_{3-j}, m'_{3-j}) \, dx'_{3-j} \, dm'_{3-j}$$

and

$$E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(\tau_{3-j} \le t < \tau_j) \, | \, \mathcal{G}_t^{3-j}] \tag{4.2}$$
$$= I(\tau_{3-j} \le t) \, E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(M_t^j > b_j) \, | \, \mathcal{G}_t^{3-j}]$$
$$= I(\tau_{3-j} \le t) \int_{b_j}^{\infty} \int_{b_j}^{\infty} H(t, x'_j, m'_j, X_t^{3-j}, M_t^{3-j}) \, g_j(x_j, m_j; t, x'_j, m'_j) \, dx'_j \, dm'_j$$

for all $0 \le t \le T$, where the functions g_j, $j = 1, 2$, are given in (2.11) above.

Now, taking into account Markovian structure of the process (X^1, M^1, X^2, M^2), we obtain:

$$E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(\tau_{3-j} \le t < \tau_j) \, | \, \mathcal{G}_t^j] \tag{4.3}$$
$$= I(t < \tau_j) \int_0^t \int_0^{\infty} \int_0^{b_{3-j}} H(t, X_t^j, M_t^j, x'_{3-j}, m'_{3-j}) \, h_{3-j}(x_{3-j}; v)$$
$$\times \, g_{3-j}(b_{3-j}, b_{3-j}; t - v, x'_{3-j}, m'_{3-j}) \, dv \, dx'_{3-j} \, dm'_{3-j}$$

and

$$E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(\tau_{3-j} < \tau_j \le t) \, | \, \mathcal{G}_t^j] \tag{4.4}$$
$$= I(\tau_j \le t) \int_0^{\tau_j} \int_0^{\infty} \int_0^{b_{3-j}} H(t, X_t^j, M_t^j, x'_{3-j}, m'_{3-j}) \, h_{3-j}(x_{3-j}; v)$$
$$\times \, g_{3-j}(b_{3-j}, b_{3-j}; \tau_j - v, x'_{3-j}, m'_{3-j}) \, dv \, dx'_{3-j} \, dm'_{3-j}$$

as well as

$$E_{x_1,m_1,x_2,m_2}[H(t, X_t^j, M_t^j, X_t^{3-j}, M_t^{3-j}) \, I(\tau_{3-j} < \tau_j \le t) \, | \, \mathcal{G}_t^{3-j}] \tag{4.5}$$
$$= I(\tau_{3-j} \le t) \int_{\tau_{3-j}}^t \int_{b_j}^{\infty} \int_{b_j}^{\infty} H(t, x'_j, m'_j, X_t^{3-j}, M_t^{3-j}) \, h_j(x_j; u)$$
$$\times \, g_j(b_j, b_j; t - u, x'_j, m'_j) \, du \, dx'_j \, dm'_j$$

for $0 \le t \le T$, and the functions g_j and h_j, $j = 1, 2$, are given in (2.11) and (2.13) above.

Summarizing the facts proved above, let us formulate the following assertion.

Proposition 4.1. *The rational price of the European contingent claim in (2.8) under partial information is given by the sum of the terms in (4.1), (4.2), (4.3), (4.4) and (4.5), where the function H is given appropriately, by the corresponding values in (3.3), (3.6), (3.9), (3.12), (3.15) or (3.20), respectively.*

5 Some examples

In this section, we derive explicit expressions for rational prices of the European exchange (Margrabe) options, in the considered model with random dividends, under full information.

In order to give an illustrating example, let us provide computations for the first term in the expression in (2.7), for the payoff function $C(s_1, s_2) = (s_1 - s_2)^+$, $s_i > 0$, $i = 1, 2$. The other terms in (2.7) can be computed similarly. It is easily seen from the structure or the processes S^i and X^i in (2.3) and (2.1) that, in that case, for each $T > 0$ and the starting values s_i, x_i, $i = 1, 2$, we have $C_0(T, y, z) = (D_0^1(T, y) - D_0^2(T, z))^+$, where $D_0^i(T, x) = s_i(x/x_i)^{\alpha_i} e^{\beta_{i,0} T}$ and $\alpha_i = \sigma_i/\theta_i$ and $\beta_{i,0} = \sigma_i\theta_i/2 - \sigma_i\eta_i/\theta_i - \sigma_i^2/2 - \delta_{i,0}$, $i = 1, 2$, as above. It thus follows from (3.1) that:

$$\begin{aligned} & E_{x_1,m_1,x_2,m_2}[(D_0^1(T, X_T^1) - D_0^2(T, X_T^2))^+ \, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \\ & = I(t < \tau_1 \wedge \tau_2) \, E_{x_1,m_1,x_2,m_2}[(D_0^1(T, X_T^1) - D_0^2(T, X_T^2))^+ \, I(T < \tau_1 \wedge \tau_2) \,|\, \mathcal{G}_t] \\ & = I(t < \tau_1 \wedge \tau_2) \, E_{X_t^1, M_t^1, X_t^2, M_t^2}[(D_0^1(T', X_{T'}^1) - D_0^2(T', X_{T'}^2))^+ \, I(T' < \tau_1' \wedge \tau_2')] \end{aligned} \tag{5.1}$$

for all $0 \le t \le T$. Hence, we have from (3.2) that:

$$\begin{aligned} & E_{x_1,m_1,x_2,m_2}[(D_0^1(T', X_{T'}^1) - D_0^2(T', X_{T'}^2))^+ \, I(T' < \tau_1' \wedge \tau_2')] \\ & = E_{x_1,m_1,x_2,m_2}[(D_0^1(T', X_{T'}^1) - D_0^2(T', X_{T'}^2)) \, I(M_{T'}^1 > b_1, M_{T'}^2 > b_2, D_0^1(T', X_{T'}^1) > D_0^2(T', X_{T'}^2))] \end{aligned} \tag{5.2}$$

for $x_i \ge m_i > b_i > 0$, $i = 1, 2$.

Let us now observe that, following the line of the arguments from [12; Theorem A.6.1], it is shown that there exists a probability measure $\widetilde{P}^i$ being locally equivalent to P on the filtration $(\mathcal{G}_t)_{t \ge 0}$ and such that its density process is given by:

$$\frac{d\widetilde{P}^i}{dP} \Big|_{\mathcal{G}_t} = \exp\Big(\sigma_i \, W_t^i - \frac{\sigma_i^2}{2} \, t \Big) \tag{5.3}$$

for all $t \geq 0$ and every $i = 1, 2$. Then, by Girsanov's theorem (see, e.g. [8; Theorem 6.3]), we may conclude that the process $\widetilde{W}^i = (\widetilde{W}^i_t)_{t \geq 0}$, defined by $\widetilde{W}^i_t = W^i_t - \sigma_i \, t$, is a standard Brownian motion under the measure $\widetilde{P}^i$. Note that, since the processes W^i, $i = 1, 2$, are assumed to be independent, the process W^{3-i} remains a standard Brownian motion under $\widetilde{P}^i$, for every $i = 1, 2$. Hence, it is seen from (2.1) that the process X^i has the expression:

$$X^i_t = x_i \, \exp\left(\left(\eta_i + \sigma_i \theta_i - \frac{\theta_i^2}{2}\right) t + \theta_i \, \widetilde{W}^i_t\right) \tag{5.4}$$

for every $i = 1, 2$. We also note that, using the explicit expression in (2.1), we obtain from (5.3) that:

$$\frac{d\widetilde{P}^i}{dP}\Big|_{\mathcal{G}_t} = e^{(\beta_{i,0} + \delta_{i,0})t} \left(\frac{X^i_t}{x_i}\right)^{\alpha_i} \tag{5.5}$$

for all $t \geq 0$.

Taking into account the structure of the processes X^i and S^i in (2.1) and (2.3), we therefore conclude from (5.1)-(5.2) and (5.5) that the expression in (3.3) takes the form:

$$\begin{aligned} &E_{x_1,m_1,x_2,m_2}[(D^1_0(T, X^1_T) - D^2_0(T, X^2_T))^+ \, I(T < \tau_1 \wedge \tau_2) \, | \, \mathcal{G}_t] = I(t < \tau_1 \wedge \tau_2) \\ &\quad \times \Big(S^1_t \, e^{-\delta_{1,0}(T-t)} \, \widetilde{P}^1_{X^1_t, M^1_t, X^2_t, M^2_t}(M^1_{T-t} > b_1, M^2_{T-t} > b_2, D^1_0(T-t, X^1_{T-t}) > D^2_0(T-t, X^2_{T-t})) \\ &\qquad - S^2_t \, e^{-\delta_{2,0}(T-t)} \, \widetilde{P}^2_{X^1_t, M^1_t, X^2_t, M^2_t}(M^1_{T-t} > b_1, M^2_{T-t} > b_2, D^1_0(T-t, X^1_{T-t}) > D^2_0(T-t, X^2_{T-t}))\Big) \end{aligned} \tag{5.6}$$

where

$$\begin{aligned} &\widetilde{P}^i_{X^1_t, M^1_t, X^2_t, M^2_t}(M^1_{T-t} > b_1, M^2_{T-t} > b_2, D^1_0(T-t, X^1_{T-t}) > D^2_0(T-t, X^2_{T-t})) \\ &= \int_{b_1}^{\infty} \int_{b_1}^{\infty} \int_{b_2}^{\infty} \int_{b_2}^{\infty} I(D^1_0(T-t, x'_1) > D^2_0(T-t, x'_2)) \prod_{\ell=1}^{2} \widetilde{g}^i_\ell(X^\ell_t, M^\ell_t; T-t, x'_\ell, m'_\ell) \, dx'_\ell \, dm'_\ell \end{aligned} \tag{5.7}$$

with the functions $\widetilde{g}^i_\ell$, $\ell = 1, 2$, defined as g_i in (2.11) above with $\widetilde{\rho}^i_i = \eta_i/\theta_i + \sigma_i - \theta_i/2$ and $\widetilde{\rho}^i_{3-i} = \eta_{3-i}/\theta_{3-i} - \theta_{3-i}/2$ in place of ρ_i, for every $i = 1, 2$.

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