Computationally Efficient Coordination in Game Trees

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Overview

Aim:

New definition of **correlated equilibrium** for game trees
(= extensive games with perfect recall),
called **EFCE** (**E**xtensive **F**orm **C**orrelated **E**quilibrium)
which is "**natural**" and **computationally tractable**

Overview:

- Example: a signalling problem
- what are correlated equilibria (CE)?
- communication and CE: the role of information sets
- define EFCE
- computational aspects

Background

R. J. Aumann (1974),
Subjectivity and correlation in randomized strategies.
Journal of Mathematical Economics **1**, 67-96.

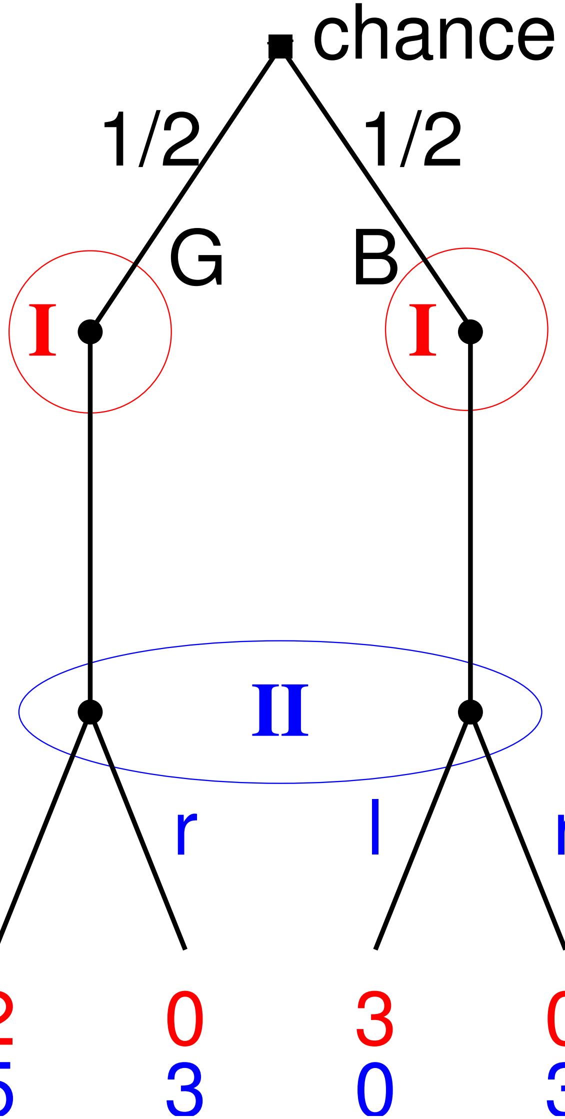
R. B. Myerson (1986), Multistage games with communication. *Econometrica* **54**, 323-358.

F. Forges (1986), An approach to communication equilibria. *Econometrica* **54**, 1375-1385.

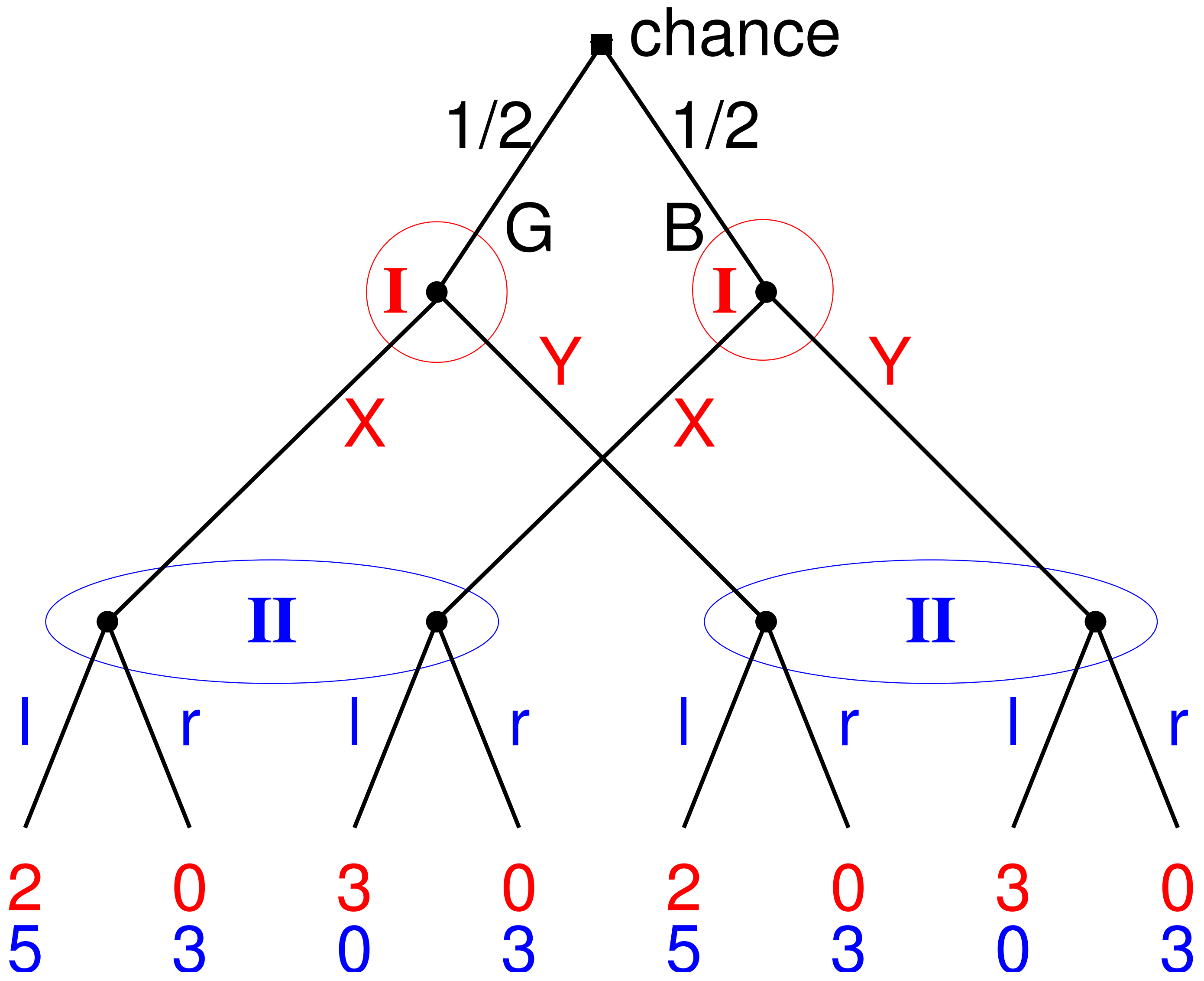
R. J. Aumann (1987), Correlated equilibrium as an expression of Bayesian rationality. *Econometrica* **55**, 1-18.

F. Forges (1993), Five legitimate definitions of correlated equilibrium in games with incomplete information. *Theory and Decision* **35**, 277-310.

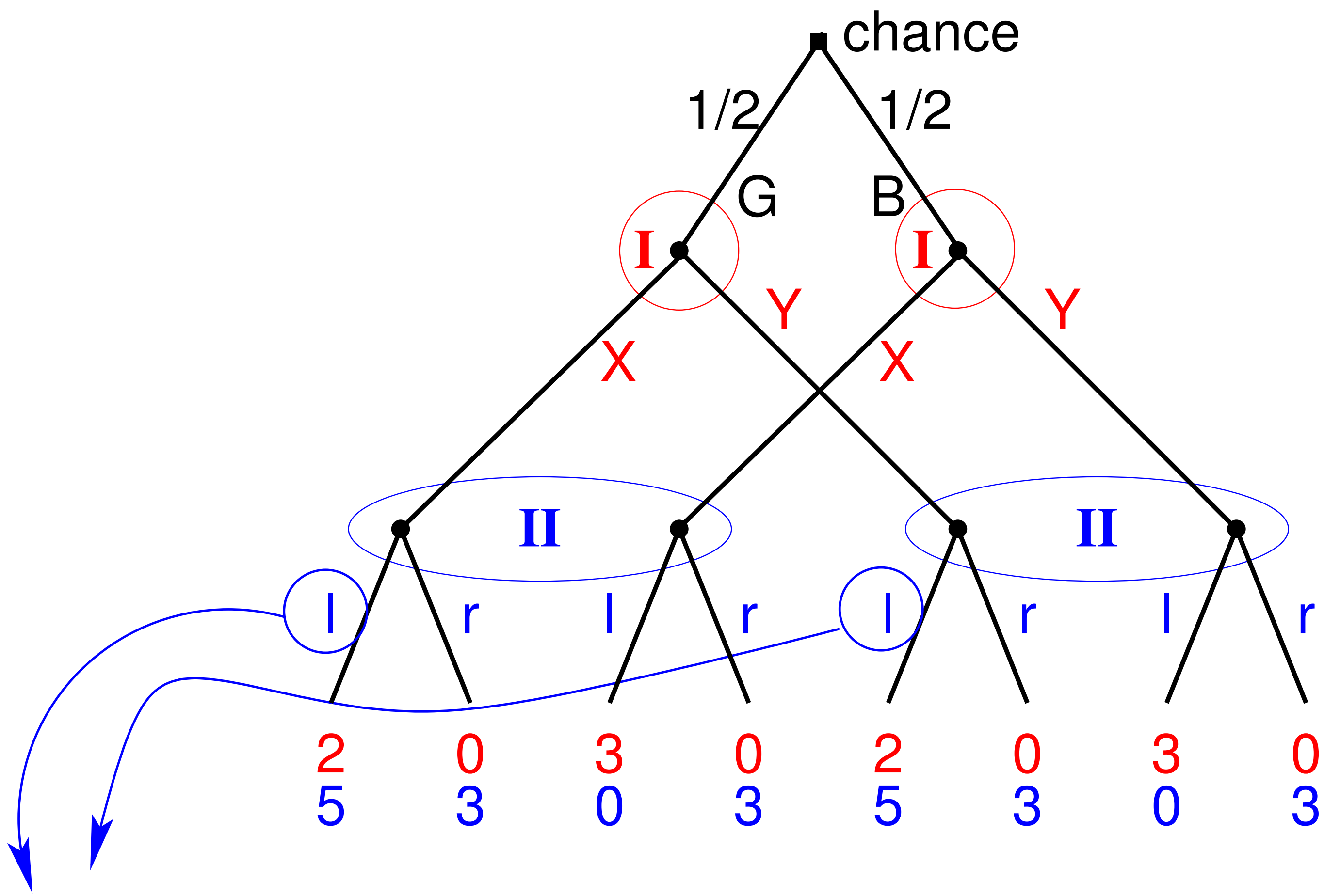
Accept a research student?



Explicit signals for player I



No type–revealing equilibria



same probabilities, otherwise both G and B prefer signal X or Y with higher acceptance chance, so one signal has prob(B) at least 1/2, l not optimal, prob(l)=0.

Goal:

Introduce concept of **coordination** in game trees

via **correlated equilibrium**,

in this example to achieve a
type-revealing equilibrium:

allowing the good (G) student and professor to coordinate

Chicken

I \ II	drive	swerve
Drive	-1 -1	0 1
Swerve	1 0	0 0

Nash equilibria

I \ II	left	right
Top	4 (I), 4 (II)	1 (I), 5 (II)
Bottom	5 (I), 1 (II)	0 (I), 0 (II)

play

0	0
1	0

pay: 1 (II), 5 (I)

play

0	1
0	0

pay: 5 (II), 1 (I)

play

1/4	1/4
1/4	1/4

pay: 2.5 (II), 2.5 (I)

Correlated equilibria

I \ II	left	right
Top	4 (I), 4 (II)	1 (I), 5 (II)
Bottom	5 (I), 1 (II)	0 (I), 0 (II)

play

0	1/2
1/2	0

pay: 3 (I), 3 (II)

play

1/3	1/3
1/3	0

pay: 3 1/3 (I), 3 1/3 (II)

play

0	1/3
1/3	1/3

pay: 2 (I), 2 (II)

Incentive constraints

I \ II	left	right
Top	4, 4	1, 5
Bottom	5, 1	0, 0

(In each cell the first number is player I's payoff and the second is player II's.)

play

a	b
c	d

$$a + b + c + d = 1$$

$$a, b, c, d \geq 0$$

$$4a + 1b \geq 5a + 0b$$

$$5c + 0d \geq 4c + 1d$$

$$\Leftrightarrow \quad b \geq a, \quad c \geq d$$

$$4a + 1c \geq 5a + 0c$$

$$5b + 0d \geq 4b + 1d$$

$$\Leftrightarrow \quad c \geq a, \quad b \geq d$$

Linear incentive constraints!

set of correlated equilibria

= polytope, defined by linear incentive constraints that compare any two strategies of a player

- variables = probabilities for strategy profiles

- holds for any number of players

- find easily CE with maximum payoff(-sum)

Canonical form

"CE = players talk beforehand, with the help of a mediator"

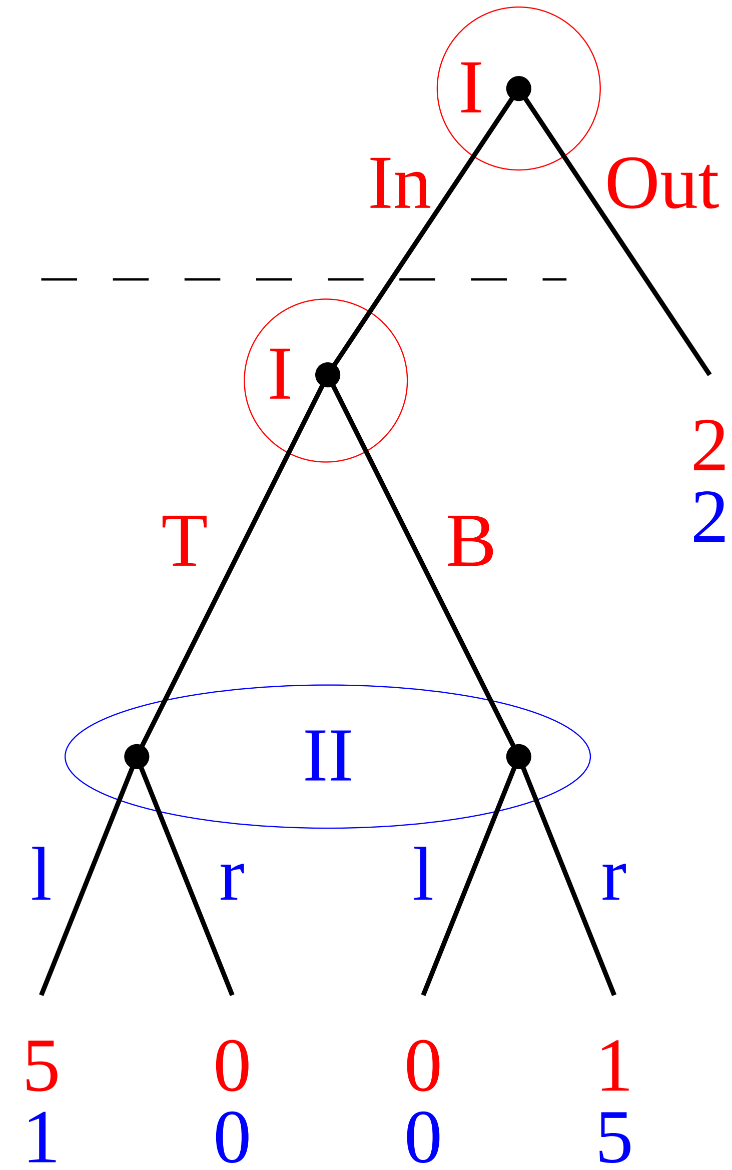
Extend game by initial stage where

- people send messages to a **device**, which computes (possibly randomly) messages, sends them back, until communication stops.
- then players act;
- look for Nash equilibrium of extended game.

Canonical form: get CE, where

- device has no inputs, uses commonly known randomization probabilities,
- messages to player = his/her strategies, followed as recommendation

New Concept? Myerson's Example



I \ II	l	r
Out	2, 2	2, 2
In, T	5, 1	0, 0
In, B	0, 0	1, 5

EFCE: use "Sealed Envelopes"

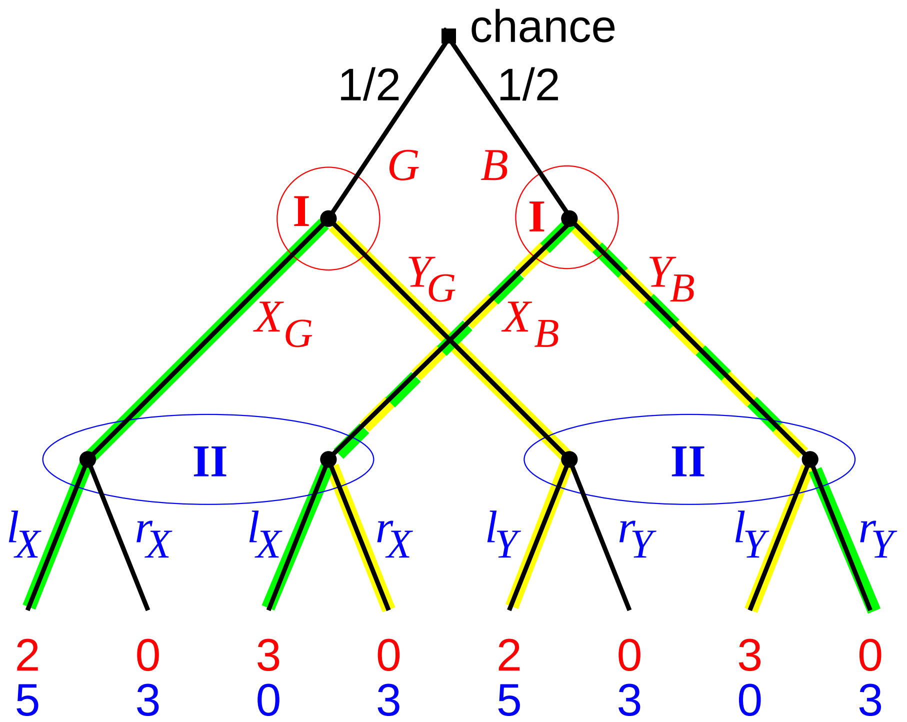
- messages generated at beginning of the game (as in the strategic form)

- **information set** enhanced with message

- player gets information at information set
  $\Rightarrow$ additional information of what to do, the recommended **move**.

- messages have to be **local**
  $\Rightarrow$ not only **delay** messages, but also **hide** them from parallel (same-stage) information sets.

Local Recommendations Only

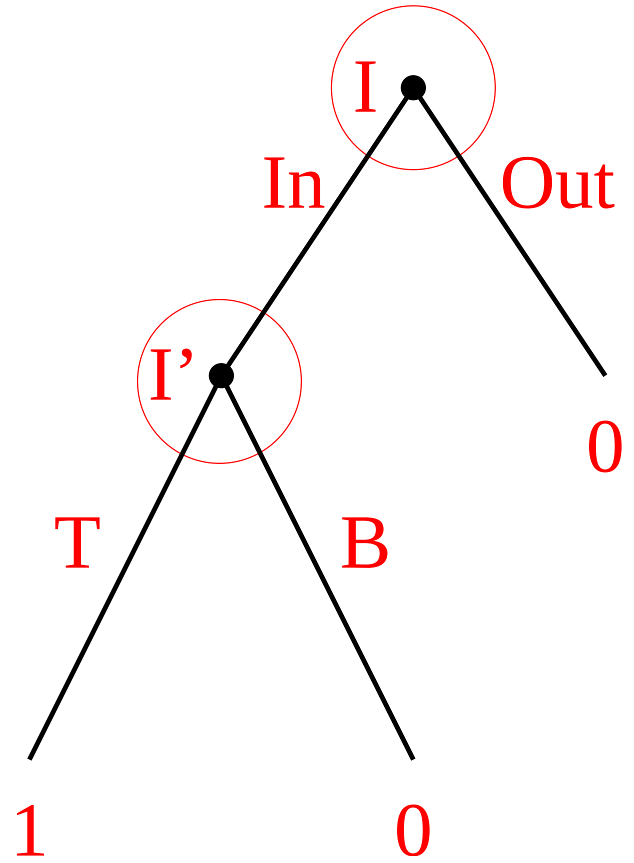


	$l_X\ l_Y$	$l_X\ r_Y$	$r_X\ l_Y$	$r_X\ r_Y$
$X_G\ X_B$	0	1/4	0	0
$X_G\ Y_B$	0	1/4	0	0
$Y_G\ X_B$	0	0	1/4	0
$Y_G\ Y_B$	0	0	1/4	0

This EFCE is **not** a normal-form CE, as B would **mimic** G.

Payoffs:	with 2 signals X,Y	with M signals X,Y,Z,...
prob 1/2:	G: 2, 5	G: 2, 5
prob 1/2:	B: (3+0)/2, (0+3)/2	B: 3/M, 3(M –1)/M
expected:	1.75, 3.25	**1**+1.5/M, **4** –1.5/M

Not the agent normal form



Note: separate issue of "perfect" CE

Extensive Form Correlated Equilibrium
EFCE

- **incentive constraints** assume

  * **average payoff** along equilibrium path
    (like agent normal form)

  * own optimization when **deviating**
    (unlike agent normal form)

$\Rightarrow$ **reduced strategic form** suffices:
no need to specify what to do when
deviating from recommended move

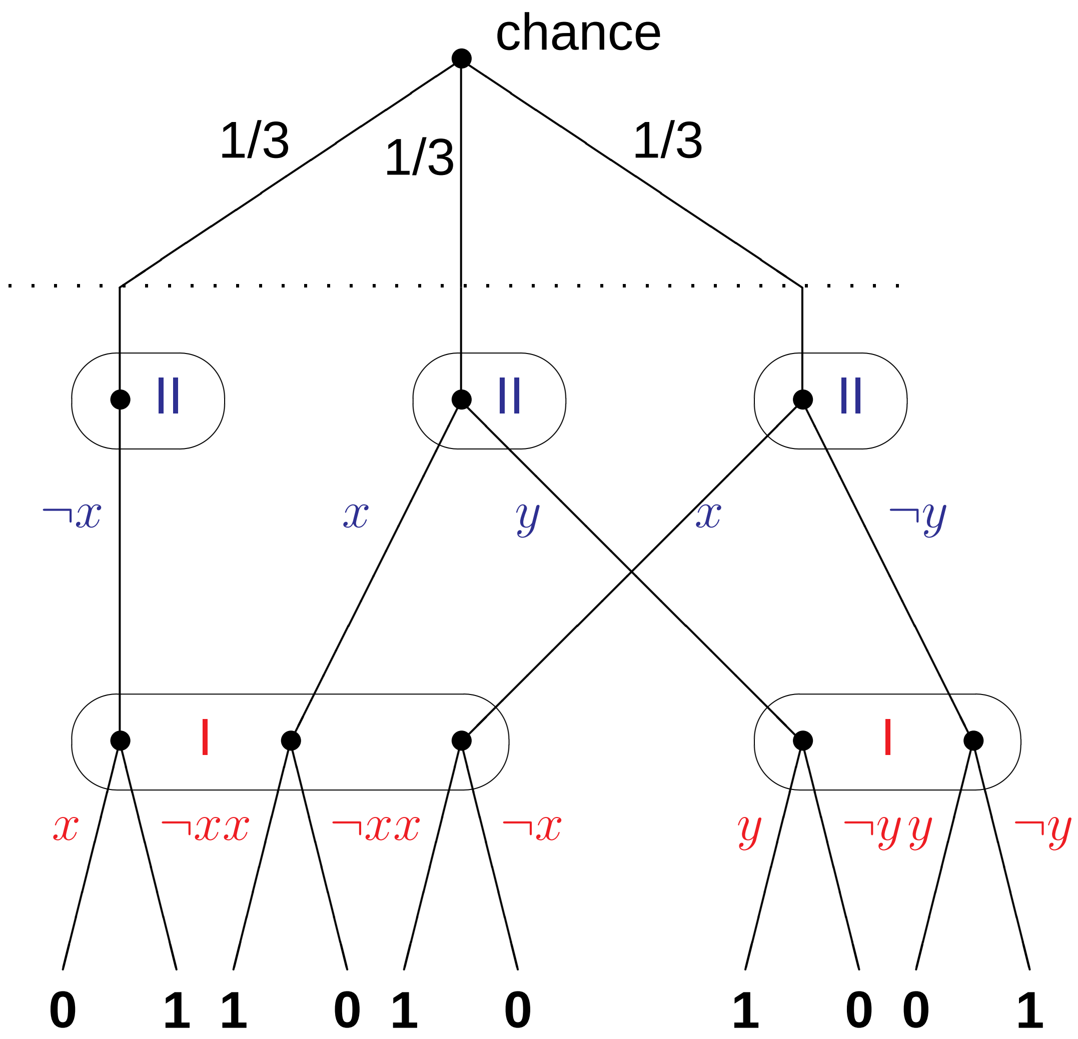
When computationally tractable?

- strategic-form CE **hard to compute** when
  2 players and chance moves, but so is any concept
  defining a convex combination of pure-strategy profiles
  (including EFCE)!

- extensive games with perfect recall:
  zero-sum 2-player games [Koller, Megiddo, BvS]:
  Nash equilibria **easy to compute**
  with **sequence form**

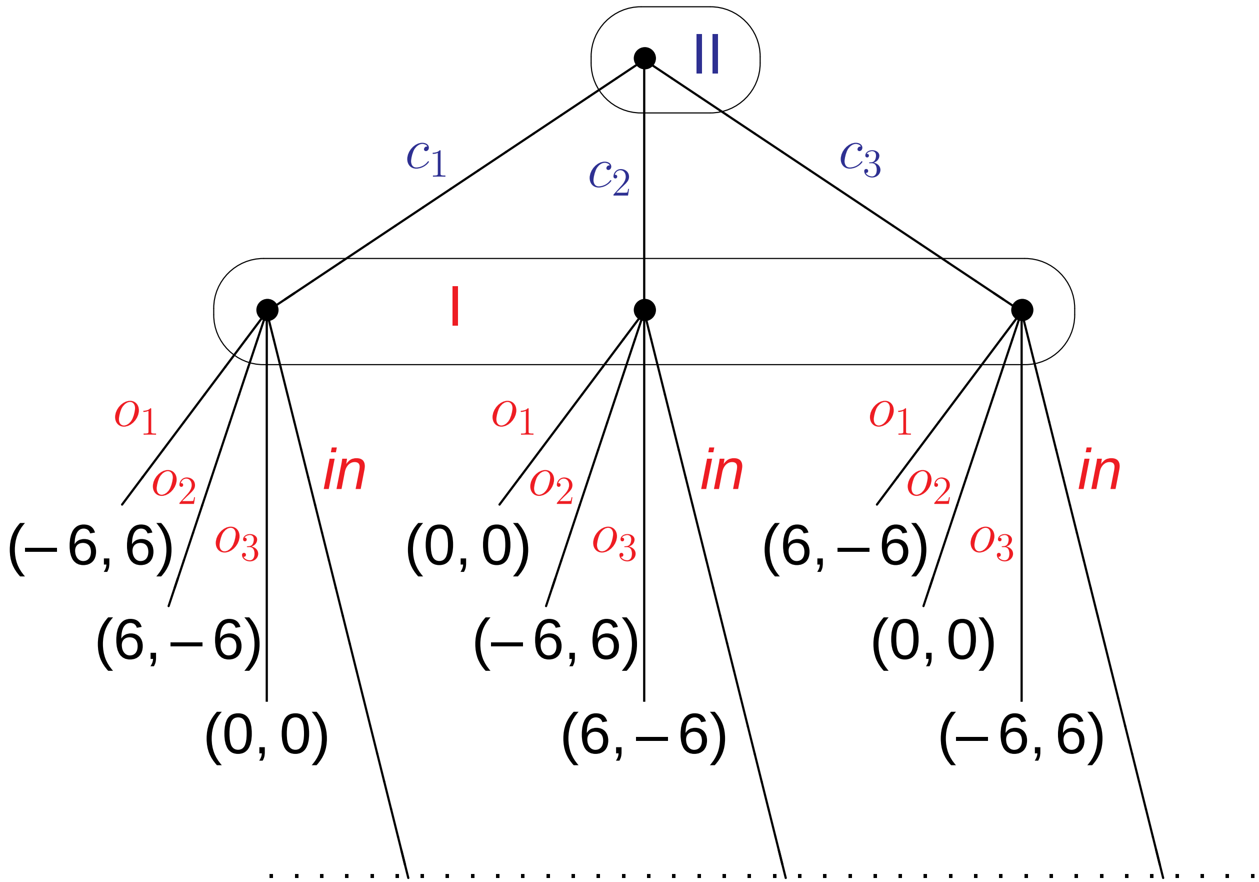
- apply sequence form to computing EFCE for
  2-player games of no chance

Strategic-form CE are NP-hard



obtained from SAT instance $(\neg x) \wedge (x \vee y) \wedge (x \vee \neg y)$

... even without chance moves



Pre-play with a zero-sum game of generalized
“rock–scissors–paper” instead of chance

Idea: Correlate Moves

- **consistency constraints**

- **incentive constraints**

- want small number (polynomial in size of game tree) of linear (in)equalities

- generate from solution a **pure strategy pair**, = moves recommended to the 2 players

Not too restrictive!

- given an own move recommendation, obtain a conditional **behavior strategy** of other player.

  (local randomization of moves, equivalent to mixed strategy if perfect recall [Kuhn])

- **need strategy** of opponent (including off-equilibrium path behavior) to decide if own recommendation good

- **consistency constraints?**

Consistency?

Cannot correlate moves at any two information sets independently:

- marginal probabilities for moves must agree

... but this does not suffice:

	a	b	c	d
L	1/2	0	1/2	0
R	0	1/2	0	1/2
S	0	1/2	1/2	0
T	1/2	0	0	1/2

locally but not globally consistent,

not a convex combination of pure strategy pairs.

Convex hull of pure strategy pairs

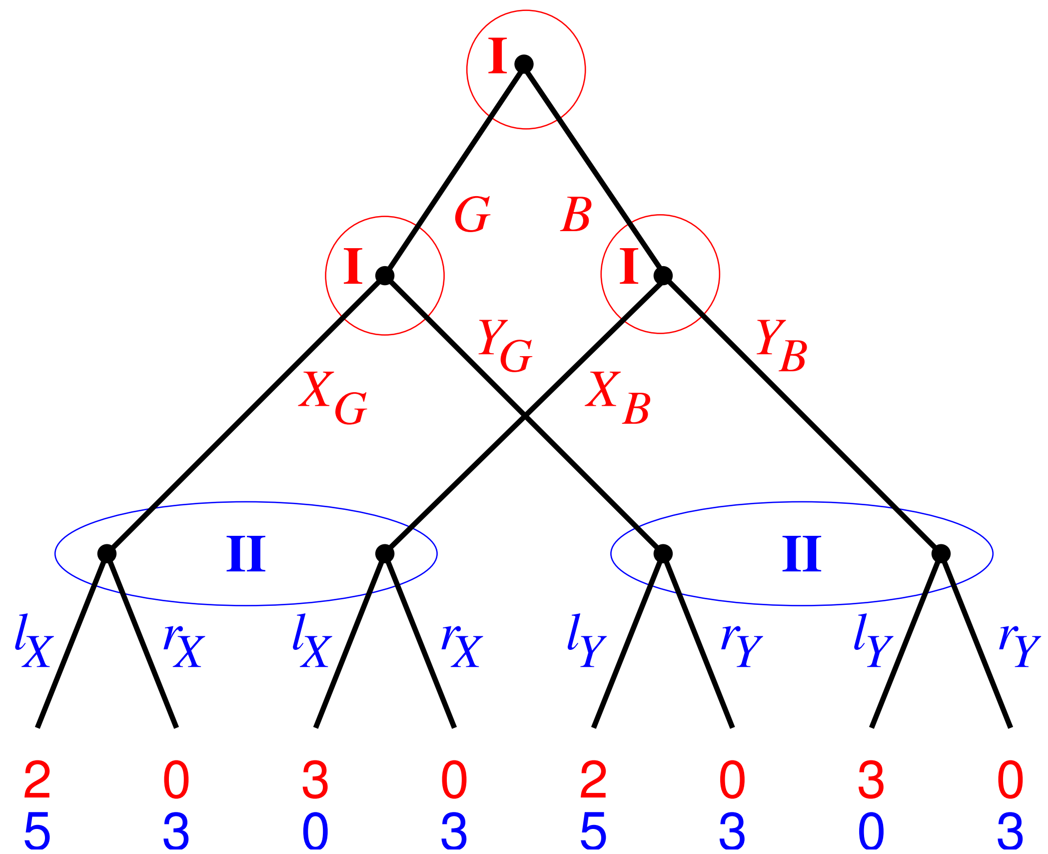
Example of pure strategy pair:

	a	b	c	d
L	1	0	1	0
R	0	0	0	0
S	0	0	0	0
T	1	0	1	0

Convex hull needs in general **exponentially many** inequalities (unless P=NP)

These arise **when there are chance moves!**

Generate move recommendations for games without chance moves



	∅	l_X	r_X	l_Y	r_Y
∅	1	1/2	1/2	1/2	1/2
G	1/2	1/4	1/4	1/4	1/4
B	1/2	1/4	1/4	1/4	1/4
$G\,X_G$	1/4	1/4	0	1/4	0
$G\,Y_G$	1/4	0	1/4	0	1/4
$B\,X_B$	1/4	0	1/4	1/4	0
$B\,Y_B$	1/4	1/4	0	0	1/4

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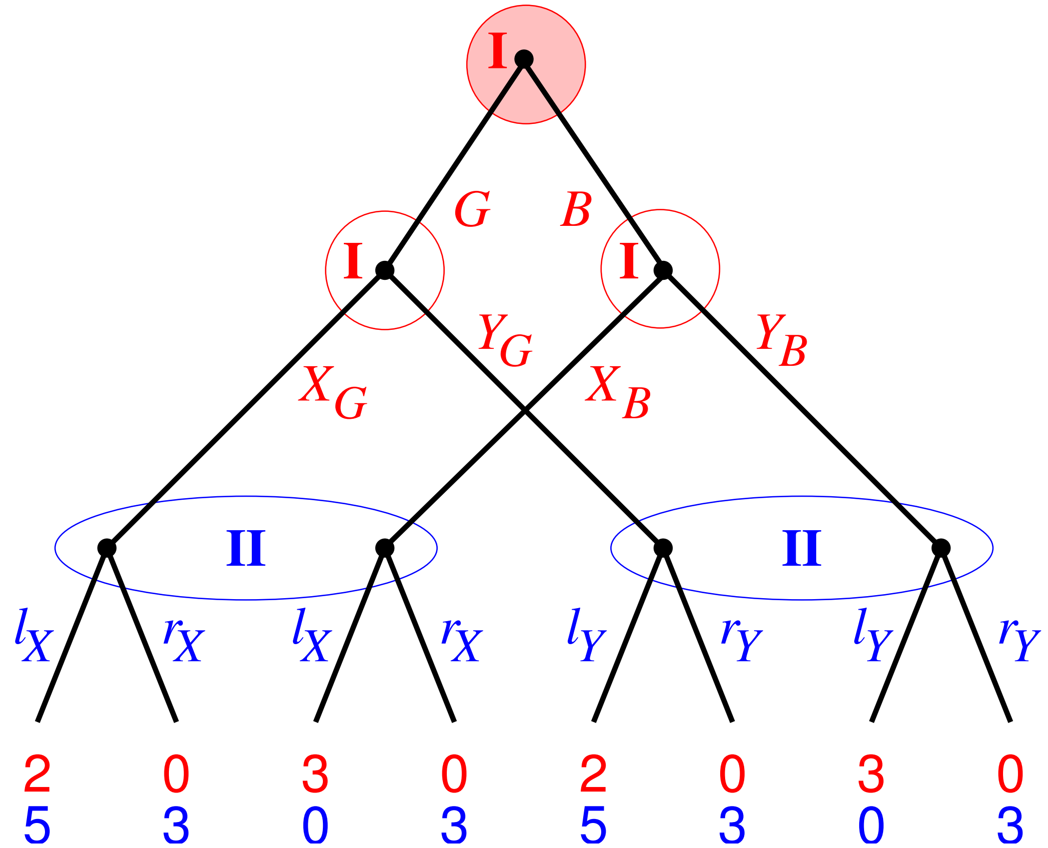

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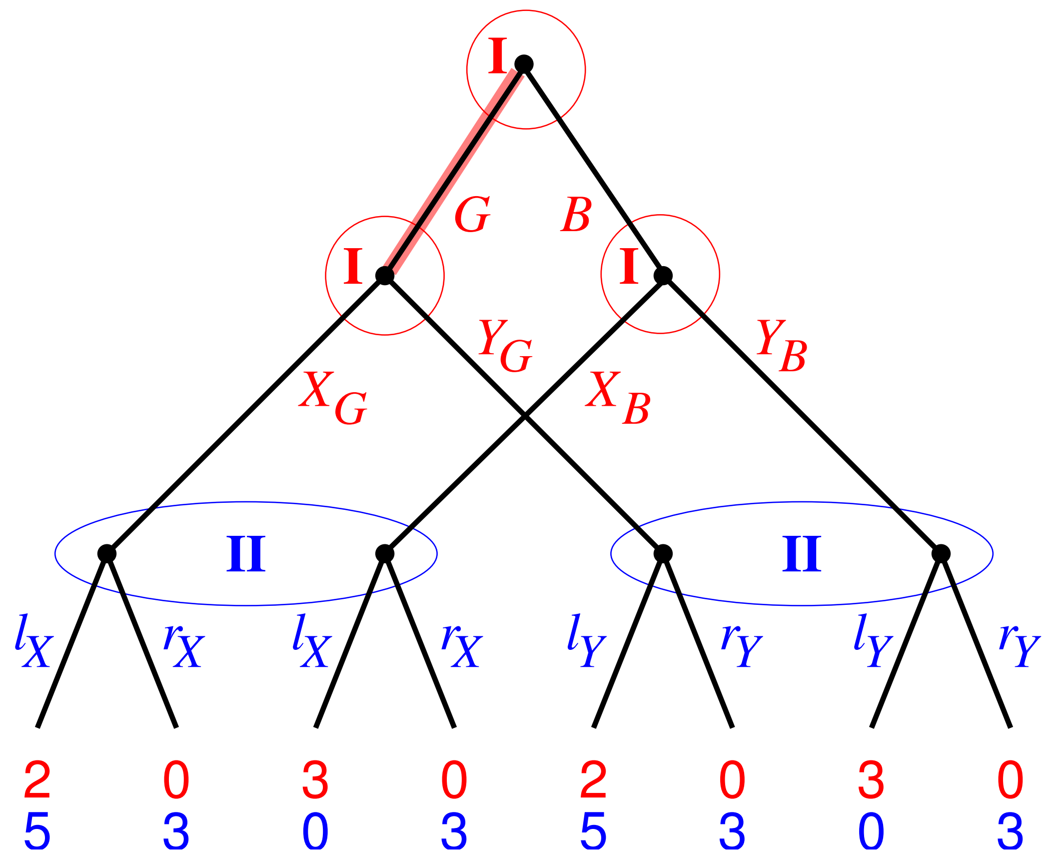

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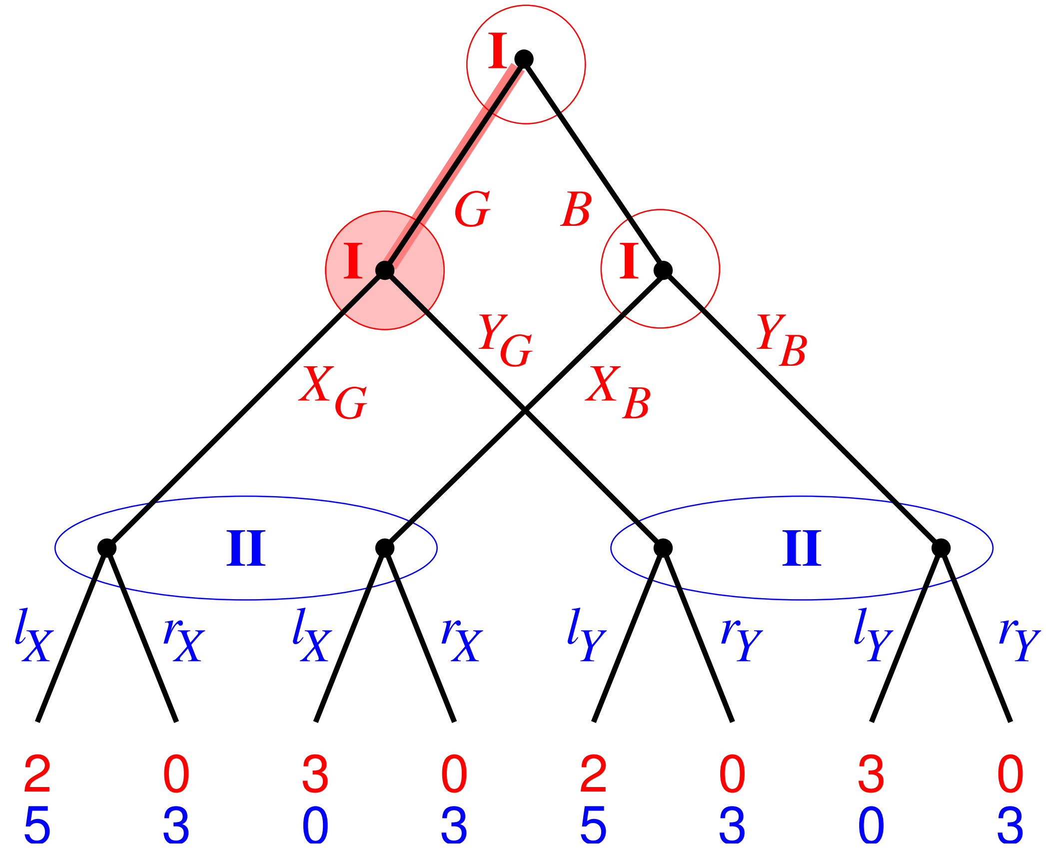

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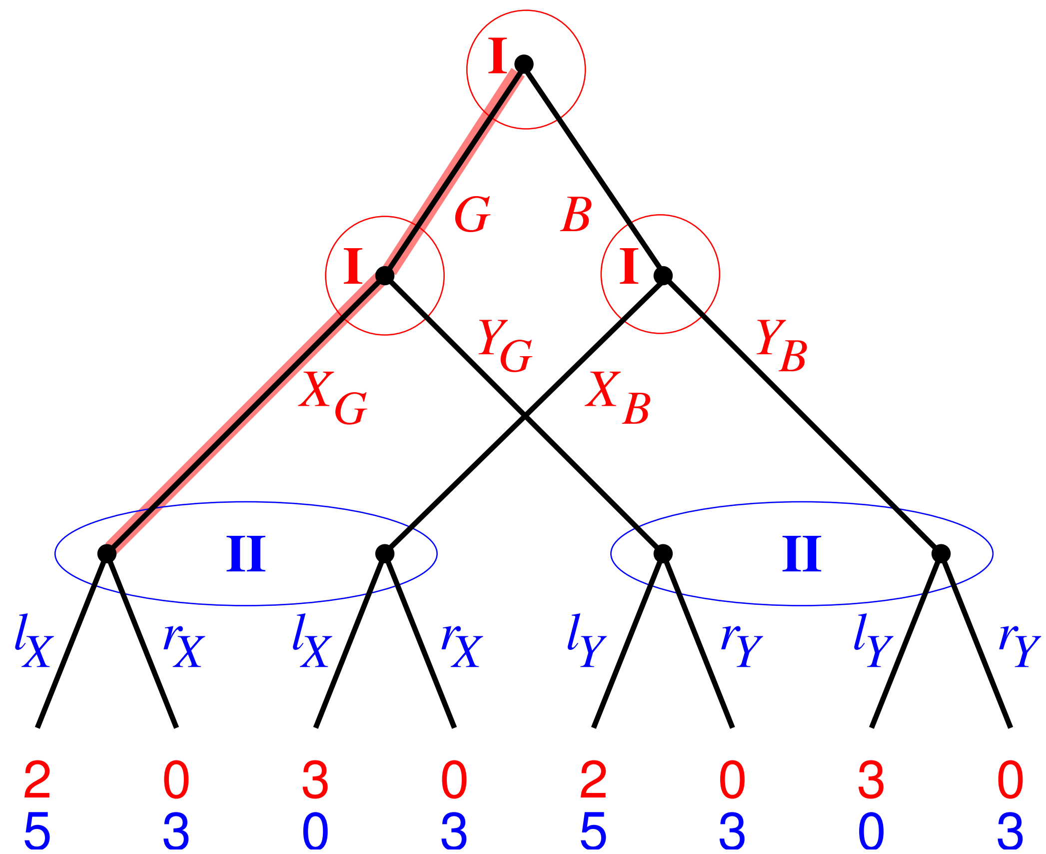

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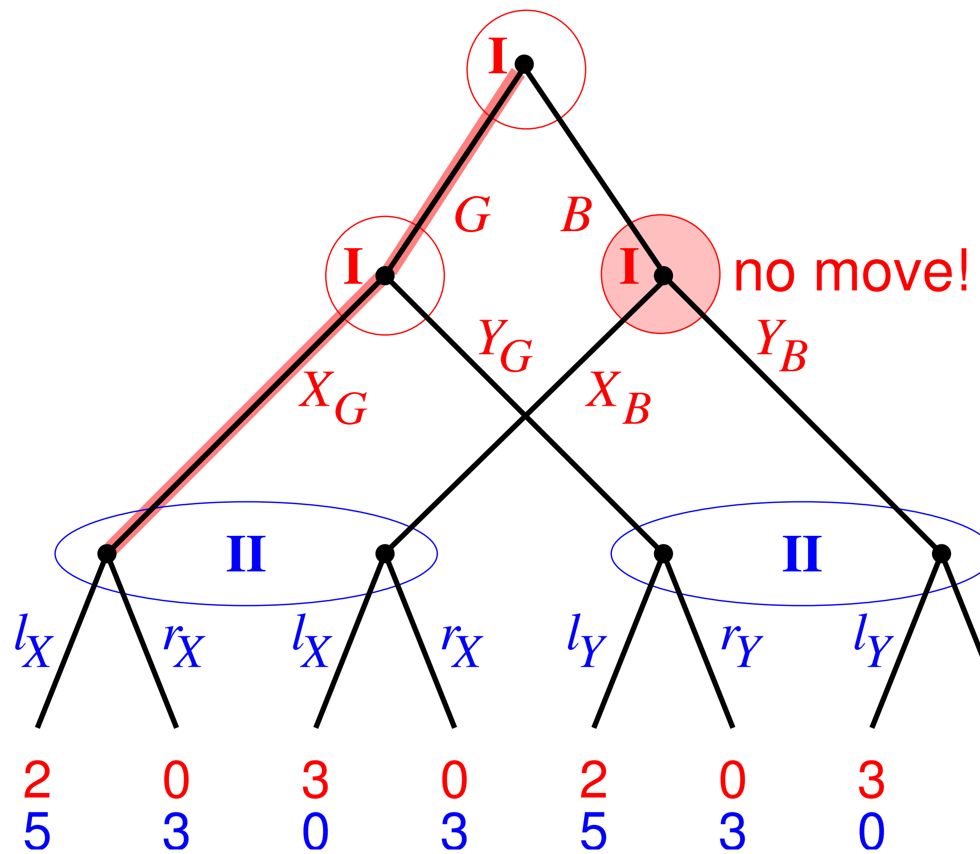

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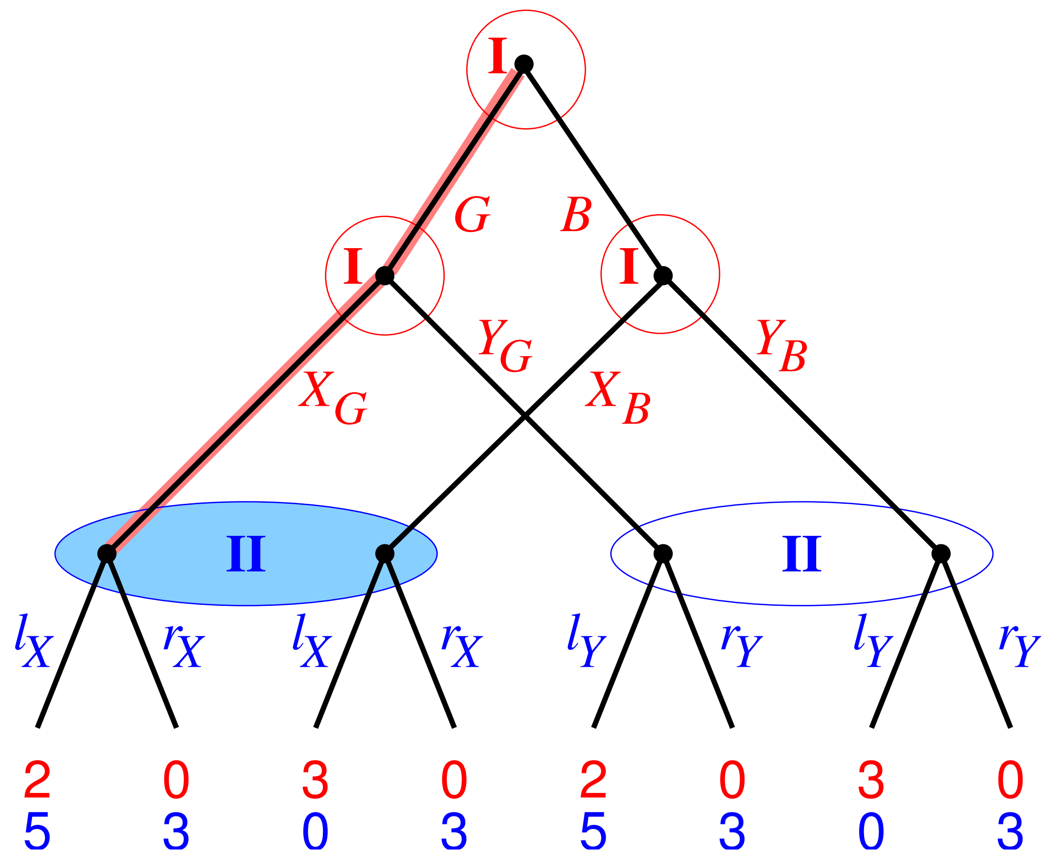

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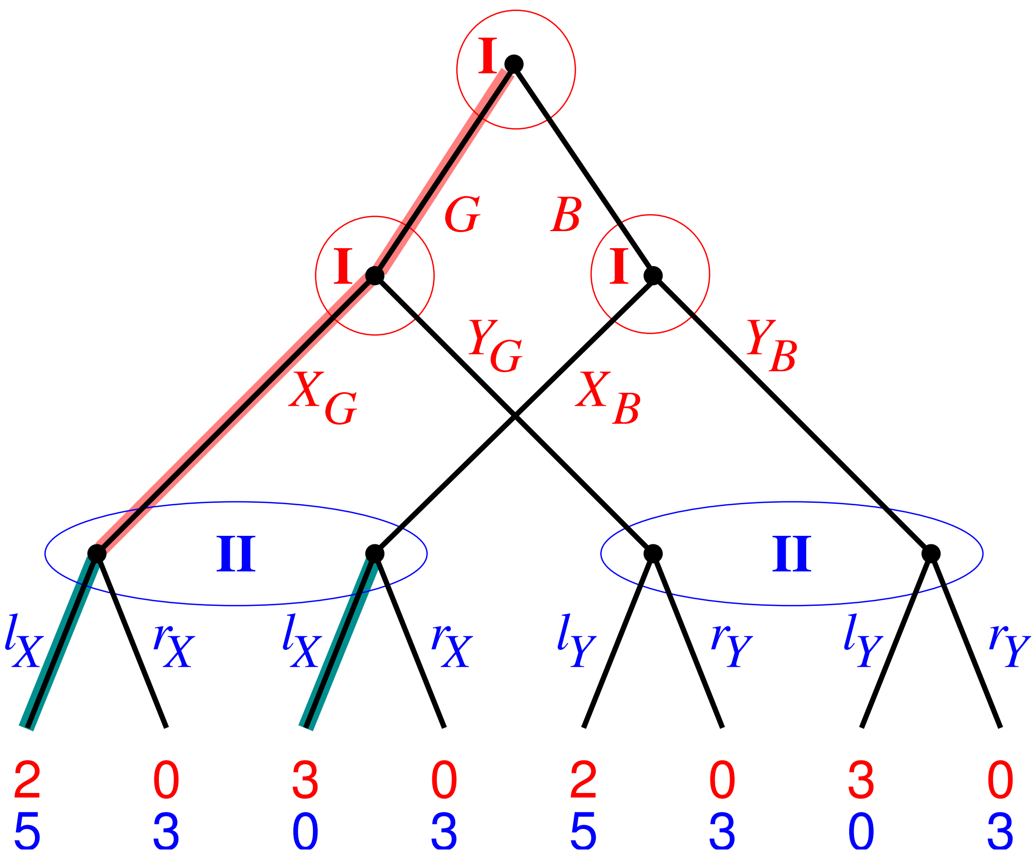

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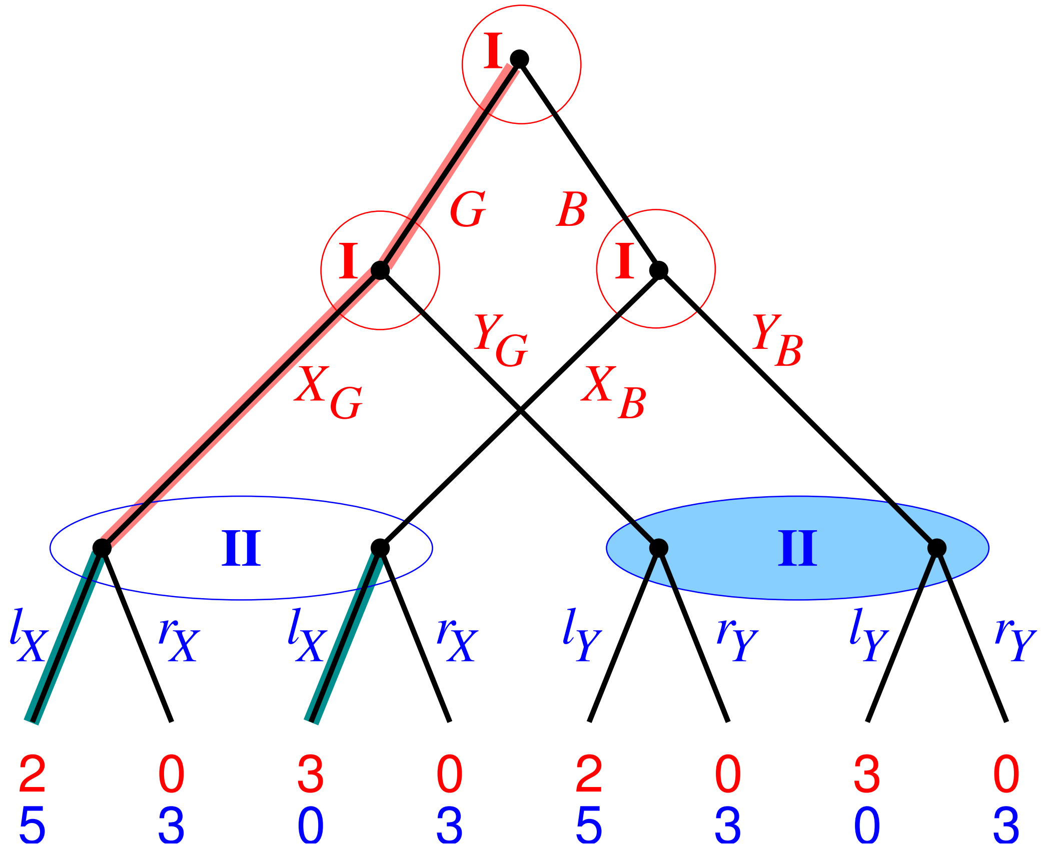

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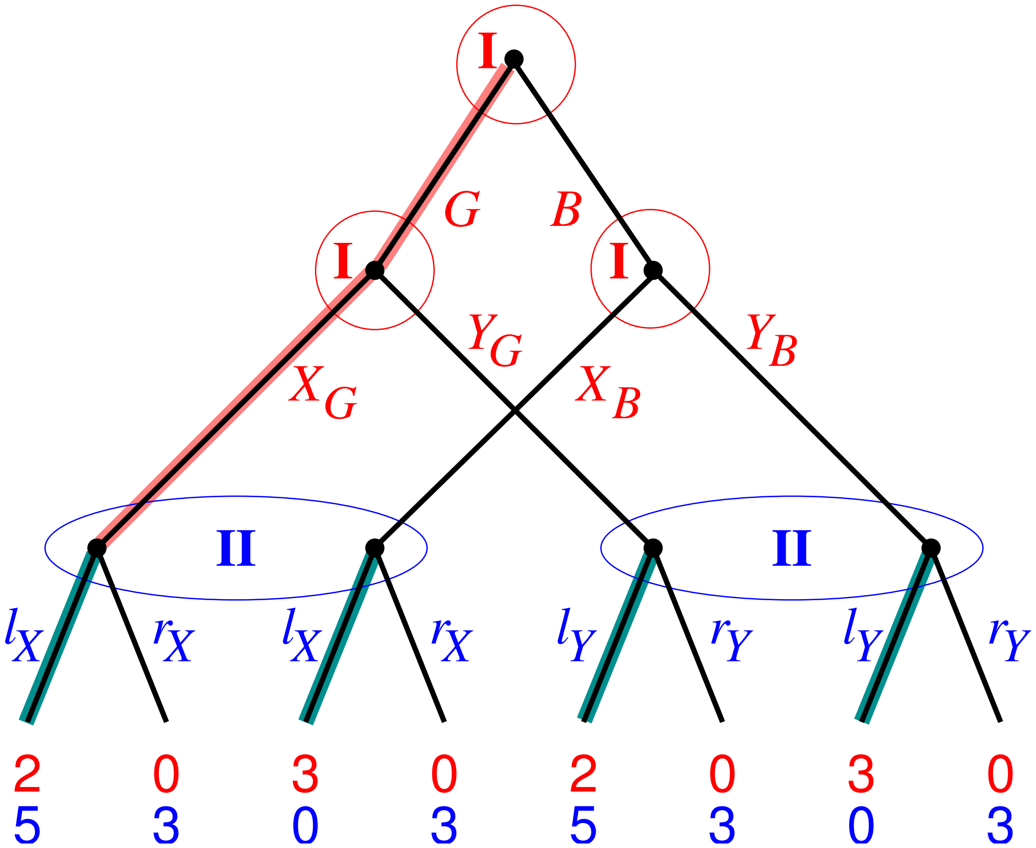

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Linear constraints

- expected payoffs **not linear** in (joint) move probabilities

- consider instead **sequences** of moves (determined by last move in sequence)

- **given** a solution fulfilling consistency and incentive constraints:

  generate corresponding pure strategy pair by top-down tree traversal, gives EFCE

  Use that 2-player games with no chance are **restrictive**: for example, have **time structure** (= know if move before or after opponent)

Incentive constraints

- **along** equilibrium path: average "own payoff"

- this payoff when following recommendation
  compared with **deviation** (alternative moves),
  optimize **dynamic-programming** style

- relatively straightforward linear inequalities.

Summary

New concept of EFCE defines

- correlated equilibrium naturally for **any** extensive game (before: only for multistage games)
- combines "**behavior strategies**" (moves instead of strategies) with **correlation**
- is **computationally tractable** for 2 players without chance moves.