

# Game Theory and Politics

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*La place d'autrui est le vray point de perspective en politique aussi bien qu'en morale.*

The **position of the other** is the true viewpoint in politics as well as in morality.

1679, Gottfried Wilhelm Leibniz (1646–1716)

$$\frac{d}{dx} \int_a^x f(s) ds = f(x)$$

## Main message

- Game theory models interaction among **multiple** “players”
- A “solution”, such as **equilibrium**, gives a recommendation to **every** player

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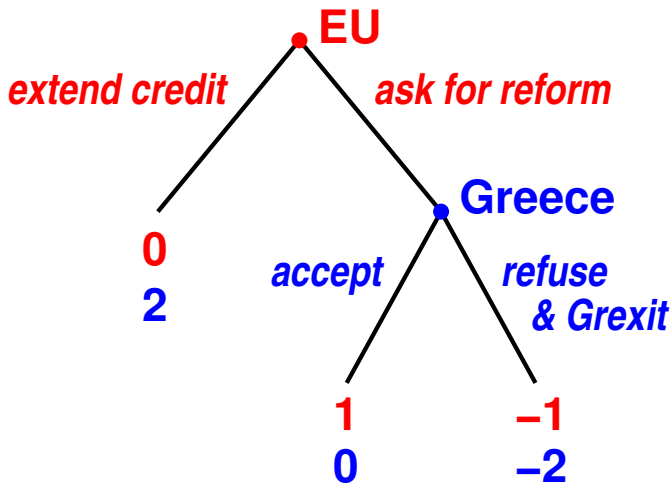
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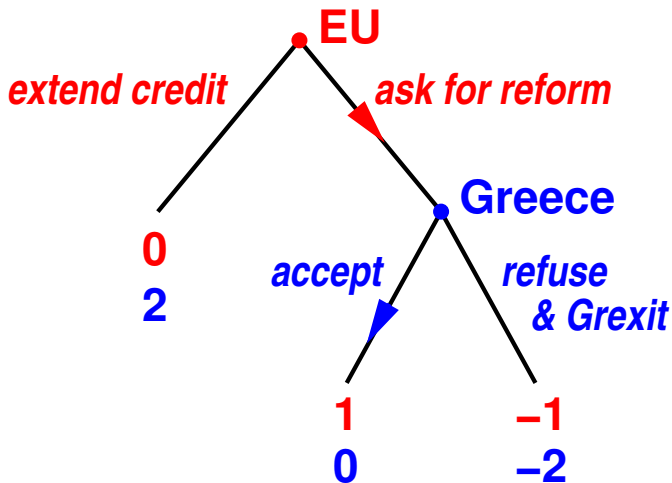
- Game theory models interaction among **multiple** “players”
- A “solution”, such as **equilibrium**, gives a recommendation to **every** player
- The **rules of the game matter**
- . . . including “unwritten” rules, conventions, habits  
(lots to do for game theory here)

# 1. Threats

## The threat game of the Greek crisis 2015

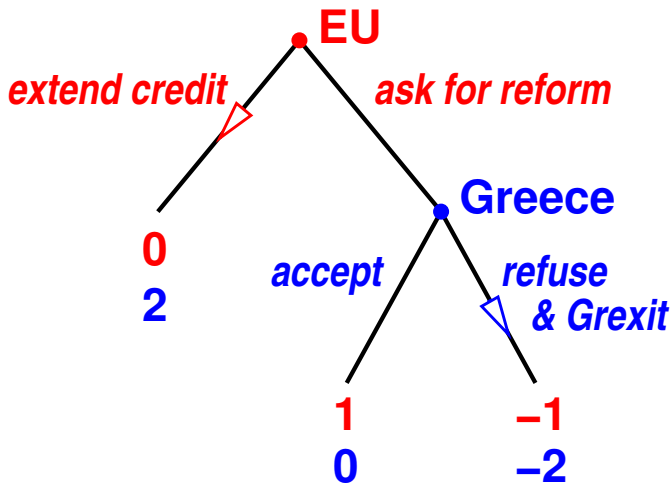


## The threat game of the Greek crisis 2015

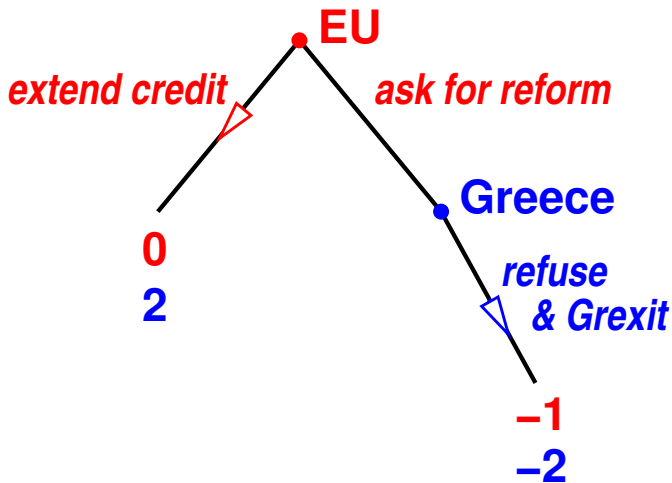




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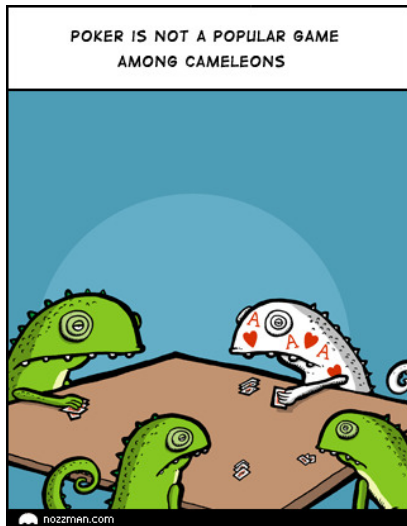
## The problem with threats

- the (ir)rationality of threats
- is the threat **credible** ?

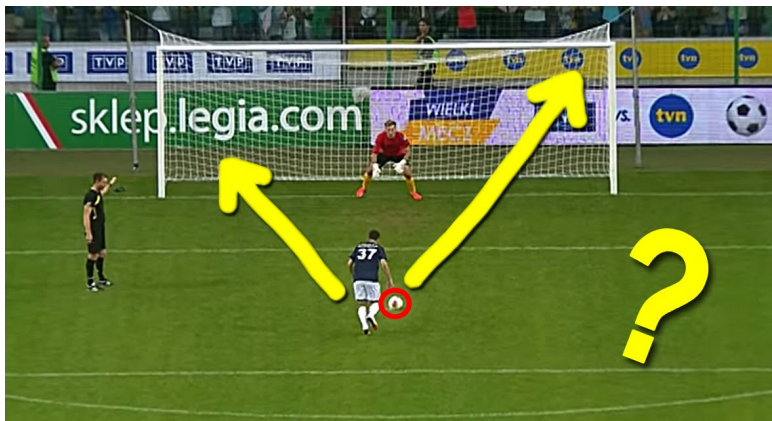
## The problem with threats

- the (ir)rationality of threats
- is the threat **credible** ?
- “call your bluff” refers to something different

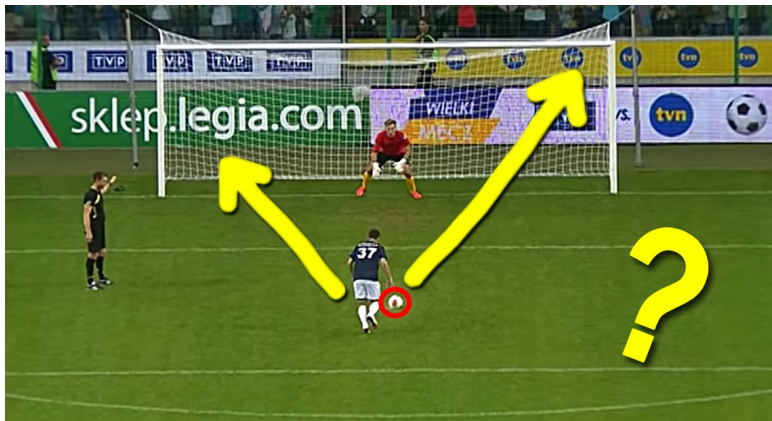
## Calling your bluff is something different



Play randomly to be unpredictable



Play randomly to be unpredictable



Unpredictability is important in **zero-sum games** with **lack of information**

## 2. (Possibly Bad) Equilibrium



## 2. (Possibly Bad) Equilibrium

- equilibrium = mutually optimal behaviour

# The Prisoner's Dilemma

		II	
		<i>C</i>	<i>D</i>
I	cooperate = <i>C</i>	2, 2	3, 0
	defect = <i>D</i>	0, 3	1, 1

# The Prisoner's Dilemma

		II	
		<i>C</i>	<i>D</i>
I	cooperate = <i>C</i>	2, 2	0, 3
	defect = <i>D</i>	3, 0	1, 1

A 2x2 payoff matrix for the Prisoner's Dilemma. The rows represent Player I's strategies: cooperate (C) and defect (D). The columns represent Player II's strategies: cooperate (C) and defect (D). The payoffs are shown in red text. A dotted diagonal line runs from the top-left cell (C,C) to the bottom-right cell (D,D).

# The Prisoner's Dilemma

		II	
		C	D
I	cooperate = C	2, 2	0, 3
	defect = D	3, 0	1, 1

A 2x2 payoff matrix for the Prisoner's Dilemma. The rows represent Player I's strategies (cooperate = C, defect = D) and the columns represent Player II's strategies (C, D). The payoffs are (I, II). The diagonal elements (2, 2) and (1, 1) are boxed in blue, while the off-diagonal elements (0, 3) and (3, 0) are boxed in red. A dotted diagonal line runs from the top-left to the bottom-right.

# The Prisoner's Dilemma

		II	
		C	D
I	cooperate = C	2, 2	0, 3
	defect = D	3, 0	1, 1

A 2x2 payoff matrix for the Prisoner's Dilemma. The rows represent Player I's strategies (cooperate = C, defect = D) and the columns represent Player II's strategies (C, D). The payoffs are shown in red text. The top-right cell (C, D) contains a blue-bordered box with the number 3. The bottom-right cell (D, D) contains a blue-bordered box with the number 1. The bottom-left cell (D, C) contains a red-bordered box with the number 3. The top-right cell (C, D) contains a red-bordered box with the number 0. A dotted diagonal line runs from the top-left to the bottom-right.

a metaphor for the “**tragedy of the commons**”

# The Prisoner's Dilemma

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a metaphor for the “**tragedy of the commons**”

... but traditional “commons” often have (unwritten) enforcement rules to prevent this tragedy

## Brinkmanship – the game of Chicken

		II	
		A	C
I	aggressive = A	0 0	1 2
	cautious = C	2 1	1 1

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		II	
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The game is symmetric (see dotted line), but its two equilibria  $(A, C)$  and  $(C, A)$  are **not** symmetric (the two players do not use the same strategy).



# The Stag Hunt game

**I**

impeach = **I**

discharge = **D**

<b>II</b>	<b>I</b>	<b>D</b>
<b>I</b>	<span style="border: 1px solid blue; padding: 2px;">2</span> , <span style="border: 1px solid red; padding: 2px;">2</span>	1, 0
<b>D</b>	0, 1	<span style="border: 1px solid blue; padding: 2px;">1</span> , <span style="border: 1px solid red; padding: 2px;">1</span>

## The Stag Hunt game

		II	
		<i>I</i>	<i>D</i>
I impeach = <i>I</i>	<i>I</i>	2, 2	1, 0
	<i>D</i>	0, 1	1, 1

discharge = *D*

Symmetric game with two symmetric equilibria (*I*, *I*) and (*D*, *D*).

What could be advantageous about (*D*, *D*)?

## The Stag Hunt game or Trust Dilemma

		II	
		<i>I</i>	<i>D</i>
I impeach = <i>I</i>	<i>I</i>	2, 2	0, 1
	<i>D</i>	1, 0	1, 1

A 2x2 payoff matrix for the Stag Hunt game. The rows represent Player I's strategies: *I* (impeach) and *D* (discharge). The columns represent Player II's strategies: *I* and *D*. The payoffs are (Player I, Player II). The top-left cell (2, 2) and bottom-right cell (1, 1) are symmetric equilibria. A dotted diagonal line runs from the top-left to the bottom-right.

Symmetric game with two symmetric equilibria (*I*, *I*) and (*D*, *D*).

What could be advantageous about (*D*, *D*)? It's risk-free!

# 3. Payoffs and Preferences

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- where do the payoff numbers come from?
- they are meant to represent a **consistent**, often just **ordinal** preference

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- where do the payoff numbers come from?
- they are meant to represent a **consistent**, often just **ordinal** preference
- are consistent preferences consistently **aggregated**?

# Maths Interlude: Brexit Numeracy

Which number is largest (UK contribution to EU budget)?

- £ 350 million per week
- £ 18.4 billion per year
- £ 5.30 per person per week

## Brexit criteria

	hard Brexit	soft Brexit	stay in the EU
Respect referendum result	<b>1</b>	<b>1</b>	<b>-1</b>
Northern Ireland peace	<b>-1</b>	<b>0</b>	<b>0</b>
Economic impact	<b>-1</b>	<b>0</b>	<b>1</b>
Sovereignty	<b>1</b>	<b>0</b>	<b>-1</b>
British world influence	<b>0</b>	<b>-1</b>	<b>1</b>



## Brexit criteria – (weighted) sum

	hard Brexit	soft Brexit	stay in the EU
Respect referendum result	<b>1</b>	<b>1</b>	<b>-1</b>
Northern Ireland peace	<b>-1</b>	<b>0</b>	<b>0</b>
Economic impact	<b>-1</b>	<b>0</b>	<b>1</b>
Sovereignty	<b>1</b>	<b>0</b>	<b>-1</b>
British world influence	<b>0</b>	<b>-1</b>	<b>1</b>
<b>total score</b>	<b>0</b>	<b>0</b>	<b>0</b>

## Brexit criteria – compare pairwise

	hard Brexit	soft Brexit	stay in the EU
Respect referendum result	1	1	-1
Northern Ireland peace	-1	0	0
Economic impact	-1	0	1
Sovereignty	1	0	-1
British world influence	0	-1	1
hard Brexit : soft Brexit			

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hard Brexit : soft Brexit	2	:	2

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hard Brexit : soft Brexit	2	:	2
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hard Brexit : soft Brexit	2	:	2
soft Brexit : stay in EU		2	:
			2

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hard Brexit : soft Brexit	<b>2</b>	<b>:</b>	<b>2</b>
soft Brexit : stay in EU		<b>2</b>	<b>:</b> <b>2</b>
hard Brexit : stay in EU			

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hard Brexit : soft Brexit	<b>2</b>	<b>:</b>	<b>2</b>
soft Brexit : stay in EU		<b>2</b>	<b>:</b> <b>2</b>
hard Brexit : stay in EU	<b>2</b>	<b>:</b>	<b>3</b>

## Brexit criteria – aggregate

	hard Brexit	soft Brexit	stay in the EU
Respect referendum result	<b>1</b>	<b>1</b>	<b>-1</b>
+ Northern Ireland peace	<b>-1</b>	<b>0</b>	<b>0</b>
= Acceptability	<b>0</b>	<b>1</b>	<b>-1</b>
Status of Britain =	<b>1</b>	<b>-1</b>	<b>0</b>
Sovereignty +	<b>1</b>	<b>0</b>	<b>-1</b>
British world influence	<b>0</b>	<b>-1</b>	<b>1</b>



## Brexit criteria – aggregate

	hard Brexit	soft Brexit	stay in the EU
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Acceptability	0	1	-1
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Status of Britain	1	-1	0
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## Brexit criteria – aggregate sum

	hard Brexit	soft Brexit	stay in the EU
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Acceptability	0	1	-1
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Economic impact	-1	0	1
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Status of Britain	1	-1	0
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total score	0	0	0
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## Brexit criteria – compare pairwise

	hard Brexit	soft Brexit	stay in the EU
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Acceptability	0	1	-1
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soft Brexit : stay in EU			1 : 2
hard Brexit : stay in EU	2	:	1

## Brexit criteria – compare pairwise

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cycles in pairwise comparisons: “**Condorcet Paradox**”

hard Brexit : soft Brexit	1	:	2
soft Brexit : stay in EU			1 : 2
hard Brexit : stay in EU	2	:	1

## 4. Strategies

## What is a strategy?

**Boris Becker:** “Strategy? I try to hit the corners of the tennis court and hope he doesn’t”

## What is a strategy?

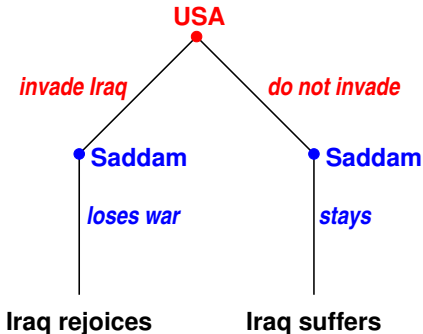
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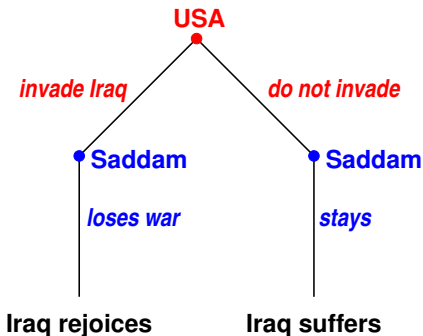
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**Game theory:**

Strategy = plan of action for every situation

## 5. The Rules of the Game Matter



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- does a “winner takes all” rule make your vote **irrelevant**?

## Quorum rule

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- A majority of votes is in favour
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### Problem

Suppose 28 % are for a strike, 20 % against

If you are against, should you vote?

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No, to avoid meeting the quorum.

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### Problem

Suppose 28 % are for a strike, 20 % against

If you are against, should you vote?

No, to avoid meeting the quorum.

Better to induce democratic participation (replace quorum rule):

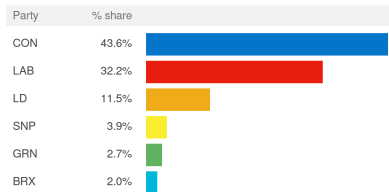
- A majority of votes is in favour
- At least 25 % (or 30 %, . . . ) members vote in favour

# First Past the Post – UK 2019 seat changes

Results analysis in maps and charts.

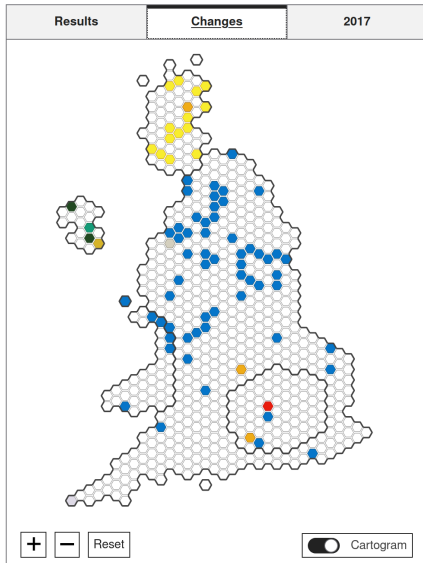
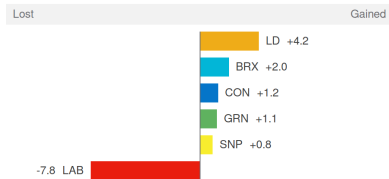
## UK vote share

After 649 of 650 seats



## UK vote share change since 2017

After 649 of 650 seats



## Is proportional representation better?

- how do **party coalitions** reflect the voters' intention?
- people are **used to** a voting system
- coalitions and referendums need practice
- learn from New Zealand

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Do voters vote **strategically**?

Your individual vote **never** matters.

⇒ There is no such thing as a “lost vote”.

⇒ voters do **not** vote strategically (and mostly not tactically).

## For discussion – challenges for game theory

- wishful thinking versus strategic thinking  
(why do so many people think only about their own tribe?)
- capture public spirit, civic duty, responsibility for the commons
- the rules of the game matter – but many unspoken rules too!

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- wishful thinking versus strategic thinking  
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Thank you!